Applied Exercises

SECS. 6.1–6.4

6.50. In certain experiments, the error made in determining the density of a substance is a random variable having a uniform density with $\alpha = -0.015$ and $\beta = 0.015$. Find the probabilities that such an error will
(a) be between -0.002 and 0.003;
(b) exceed 0.005 in absolute value.

6.51. A point D is chosen on the line AB, whose midpoint is C and whose length is a. If X, the distance from D to A, is a random variable having the uniform density with $\alpha = 0$ and $\beta = a$, what is the probability that AD, BD, and AC will form a triangle?

6.52. In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

6.53. If a company employs n salespersons, its gross sales in thousands of dollars may be regarded as a random variable having a gamma distribution with $\alpha = 80\sqrt{n}$ and $\beta = 2$. If the sales cost is \$8,000 per salesperson, how many salespersons should the company employ to maximize the expected profit?

6.54. The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with $\theta = 120$ days. Find the probabilities that such a watch will
(a) have to be reset in less than 24 days;
(b) not have to be reset in at least 180 days.

6.55. The mileage (in thousands of miles) that car owners get with a certain kind of radial tire is a random variable having an exponential distribution with $\theta = 40$. Find the probabilities that one of these tires will last
(a) at least 20,000 miles;
(b) at most 30,000 miles.

6.56. The number of bad checks that a bank receives during a 5-hour business day is a Poisson random variable with $\lambda = 2$. What is the probability that it will not receive a bad check on any one day during the first 2 hours of business?

6.57. The number of planes arriving per day at a small private airport is a random variable having a Poisson distribution with $\lambda = 28.8$. What is the probability that the time between two such arrivals is at least 1 hour?

6.58. If the annual proportion of erroneous income tax returns filed with the IRS can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 10 percent erroneous returns?

6.59. A certain kind of appliance requires repairs on the average once every 2 years. Assuming that the times between repairs are exponentially distributed, what is the probability that such an appliance will work at least 3 years without requiring repairs?

6.60. If the annual proportion of new restaurants that fail in a given city may be looked upon as a random variable having a beta distribution with $\alpha = 1$ and $\beta = 4$, find
(a) the mean of this distribution, that is, the annual proportion of new restaurants that can be expected to fail in the given city;
(b) the probability that at least 25 percent of all new restaurants will fail in the given city in any one year.

6.61. Suppose that the service life in hours of a semiconductor is a random variable having a Weibull distribution (see Exercise 6.23) with $\alpha = 0.025$ and $\beta = 0.500$.
(a) How long can such a semiconductor be expected to last?
(b) What is the probability that such a semiconductor will still be in operating condition after 4,000 hours?

SECS. 6.5–6.7

6.62. If Z is a random variable having the standard normal distribution, find
(a) $P(Z < 1.33)$;
(b) $P(Z \geqq -0.79)$;
(c) $P(0.55 < Z < 1.22)$;
(d) $P(-1.90 \leq Z \leq 0.44)$.

6.63. If Z is a random variable having the standard normal distribution, find the probabilities that it will take on a value
(a) greater than 1.14;
(b) greater than -0.36;
(c) between -0.46 and -0.09;
(d) between -0.58 and 1.12.

6.64. If Z is a random variable having the standard normal distribution, find the respective values z_1, z_2, z_3, and z_4 such that
(a) $P(0 < Z < z_1) = 0.4306$;
(b) $P(Z \geqq z_2) = 0.7704$;
(c) $P(Z > z_3) = 0.2912$;
(d) $P(-z_4 \leqq Z < z_4) = 0.9700$.

6.65. Find z if the standard-normal-curve area
(a) between 0 and z is 0.4726;
(b) to the left of z is 0.9868;

(c) to the right of z is 0.1314;
(d) between $-z$ and z is 0.8502.

6.66. If X is a random variable having a normal distribution, what are the probabilities of getting a value
(a) within one standard deviation of the mean;
(b) within two standard deviations of the mean;
(c) within three standard deviations of the mean;
(d) within four standard deviations of the mean?

6.67. If z_α is defined by

$$\int_{z_\alpha}^{\infty} n(z; 0, 1)\, dz = \alpha$$

find its values for
(a) $\alpha = 0.05$;
(b) $\alpha = 0.025$;
(c) $\alpha = 0.01$;
(d) $\alpha = 0.005$.

6.68. (a) Use a computer program to find the probability that a random variable having the normal distribution with the mean -1.786 and the standard deviation 1.0416 will assume a value between -2.159 and 0.5670.
(b) Interpolate in Table III to find this probability and compare your result with the more exact value found in part (a).

6.69. (a) Use a computer program to find the probability that a random variable having the normal distribution with mean 5.853 and the standard deviation 1.361 will assume a value greater than 8.625.
(b) Interpolate in Table III to find this probability and compare your result with the more exact value found in part (a).

6.70. Suppose that during periods of meditation the reduction of a person's oxygen consumption is a random variable having a normal distribution with $\mu = 37.6$ cc per minute and $\sigma = 4.6$ cc per minute. Find the probabilities that during a period of meditation a person's oxygen consumption will be reduced by
(a) at least 44.5 cc per minute;
(b) at most 35.0 cc per minute;
(c) anywhere from 30.0 to 40.0 cc per minute.

6.71. In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with $\mu = 15.40$ seconds and $\sigma = 0.48$ second. Find the probabilities that the time it takes to develop one of the prints will be
(a) at least 16.00 seconds;
(b) at most 14.20 seconds;
(c) anywhere from 15.00 to 15.80 seconds.

6.72. A random variable has a normal distribution with $\sigma = 10$. If the probability that the random variable will take on a value less than 82.5 is 0.8212, what is the probability that it will take on a value greater than 58.3?

6.73. Suppose that the actual amount of instant coffee that a filling machine puts into "6-ounce" jars is a random variable having a normal distribution with $\sigma = 0.05$ ounce. If only 3 percent of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars?

6.74. Check in each case whether the normal approximation to the binomial distribution may be used according to the rule of thumb on page 192.
(a) $n = 16$ and $\theta = 0.20$;
(b) $n = 65$ and $\theta = 0.10$;
(c) $n = 120$ and $\theta = 0.98$.

6.75. Suppose that we want to use the normal approximation to the binomial distribution to determine $b(1; 150, 0.05)$.
(a) Based on the rule of thumb on page 192, would we be justified in using the approximation?
(b) Make the approximation and round to four decimals.
(c) If a computer printout shows that $b(1; 150, 0.05) = 0.0036$ rounded to four decimals, what is the percentage error of the approximation obtained in part (b)?

This serves to illustrate that the rule of thumb is just that and no more; making approximations like this also requires a good deal of professional judgment.

6.76. Use the normal approximation to the binomial distribution to determine (to four decimals) the probability of getting 7 heads and 7 tails in 14 flips of a balanced coin. Also refer to Table I on pages 487–491 to find the error of this approximation.

6.77. With reference to Exercise 6.75, show that the Poisson distribution would have yielded a better approximation.

6.78. If 23 percent of all patients with high blood pressure have bad side effects from a certain kind of medicine, use the normal approximation to find the probability that among 120 patients with high blood pressure treated with this medicine more than 32 will have bad side effects.

6.79. If the probability is 0.20 that a certain bank will refuse a loan application, use the normal approximation to determine (to three decimals) the probability that the bank will refuse at most 40 of 225 loan applications.

6.80. To illustrate the law of large numbers (see also Exercise 5.54 on page 172), use the normal approximation to the binomial distribution to determine the probabilities that the proportion of heads will be anywhere from 0.49 to 0.51 when a balanced coin is flipped

$$E(X) = \mu + E(E_1) + E(E_2) + \cdots + E(E_n) = \mu$$

In other words, we are assuming that the random errors have a mean of zero, at least in the long run. We also can write

$$\text{var}(X) = (\mu + E_1 + E_2 + \cdots + E_n) = n\sigma^2$$

In other words, the variance of the *sum* of the random errors is $n\sigma^2$.

It follows that $\overline{X} = \mu + \overline{E}$, where $\overline{E}$ is the sample mean of the errors $E_1, E_2, \ldots, E_n$, and $\sigma^2_{\overline{X}} = \sigma^2/n$. The central limit theorem given by Theorem 8.3 allows us to conclude that

$$Z = \frac{\overline{X} - \mu}{\sigma\sqrt{n}}$$

is a random variable whose distribution as $n \to \infty$ is the standard normal distribution.

It is not difficult to see from this argument that most repeated measurements of the same phenomenon are, at least approximately, normally distributed. It is this conclusion that underscores the importance of the chi-square, t, and F distributions in applications that are based on the assumption of data from normally distributed populations. It also demonstrates why the normal distribution is of major importance in statistics.

Applied Exercises — SECS. 8.1–8.3

In the following exercises it is assumed that all samples are drawn without replacement unless otherwise specified.

8.60. How many different samples of size $n = 3$ can be drawn from a finite population of size

(a) $N = 12$; **(b)** $N = 20$; **(c)** $N = 50$?

8.61. What is the probability of each possible sample if

(a) a random sample of size $n = 4$ is to be drawn from a finite population of size $N = 12$;

(b) a random sample of size $n = 5$ is to be drawn from a finite population of size $N = 22$?

8.62. If a random sample of size $n = 3$ is drawn from a finite population of size $N = 50$, what is the probability that a particular element of the population will be included in the sample?

8.63. For random samples from an infinite population, what happens to the standard error of the mean if the sample size is

(a) increased from 30 to 120;

(b) increased from 80 to 180;

(c) decreased from 450 to 50;

(d) decreased from 250 to 40?

8.64. Find the value of the finite population correction factor $\frac{N-n}{N-1}$ for

(a) $n = 5$ and $N = 200$;

(b) $n = 50$ and $N = 300$;

(c) $n = 200$ and $N = 800$.

8.65. A random sample of size $n = 100$ is taken from an infinite population with the mean $\mu = 75$ and the variance $\sigma^2 = 256$.

(a) Based on Chebyshev's theorem, with what probability can we assert that the value we obtain for $\overline{X}$ will fall between 67 and 83?

(b) Based on the central limit theorem, with what probability can we assert that the value we obtain for $\overline{X}$ will fall between 67 and 83?

8.66. A random sample of size $n = 81$ is taken from an infinite population with the mean $\mu = 128$ and the standard deviation $\sigma = 6.3$. With what probability can we assert that the value we obtain for $\overline{X}$ will not fall between 126.6 and 129.4 if we use

(a) Chebyshev's theorem;

(b) the central limit theorem?

8.67. Rework part (b) of Exercise 8.66, assuming that the population is not infinite but finite and of size $N = 400$.

8.68. A random sample of size $n = 225$ is to be taken from an exponential population with $\theta = 4$. Based on the central limit theorem, what is the probability that the mean of the sample will exceed 4.5?

8.69. A random sample of size $n = 200$ is to be taken from a uniform population with $\alpha = 24$ and $\beta = 48$. Based on the central limit theorem, what is the probability that the mean of the sample will be less than 35?

8.70. A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of the sample will
(a) exceed 52.9;
(b) fall between 50.5 and 52.3;
(c) be less than 50.6?

8.71. A random sample of size 100 is taken from a normal population with $\sigma = 25$. What is the probability that the mean of the sample will differ from the mean of the population by 3 or more either way?

8.72. Independent random samples of sizes 400 are taken from each of two populations having equal means and the standard deviations $\sigma_1 = 20$ and $\sigma_2 = 30$. Using Chebyshev's theorem and the result of Exercise 8.2, what can we assert with a probability of at least 0.99 about the value we will get for $\overline{X}_1 - \overline{X}_2$? (By "independent" we mean that the samples satisfy the conditions of Exercise 8.2.)

8.73. Assume that the two populations of Exercise 8.72 are normal and use the result of Exercise 8.3 to find k such that

$$P(-k < \overline{X}_1 - \overline{X}_2 < k) = 0.99$$

8.74. Independent random samples of sizes $n_1 = 30$ and $n_2 = 50$ are taken from two normal populations having the means $\mu_1 = 78$ and $\mu_2 = 75$ and the variances $\sigma_1^2 = 150$ and $\sigma_2^2 = 200$. Use the results of Exercise 8.3 to find the probability that the mean of the first sample will exceed that of the second sample by at least 4.8.

8.75. The actual proportion of families in a certain city who own, rather than rent, their home is 0.70. If 84 families in this city are interviewed at random and their responses to the question of whether they own their home are looked upon as values of independent random variables having identical Bernoulli distributions with the parameter $\theta = 0.70$, with what probability can we assert that the value we obtain for the sample proportion $\hat{\Theta}$ will fall between 0.64 and 0.76, using the result of Exercise 8.4 and
(a) Chebyshev's theorem;
(b) the central limit theorem?

8.76. The actual proportion of men who favor a certain tax proposal is 0.40 and the corresponding proportion for women is 0.25; $n_1 = 500$ men and $n_2 = 400$ women are interviewed at random, and their individual responses are looked upon as the values of independent random variables having Bernoulli distributions with the respective parameters $\theta_1 = 0.40$ and $\theta_2 = 0.25$. What can we assert, according to Chebyshev's theorem, with a probability of at least 0.9375 about the value we will get for $\hat{\Theta}_1 - \hat{\Theta}_2$, the difference between the two sample proportions of favorable responses? Use the result of Exercise 8.5.

SECS. 8.4–8.6

(In Exercises 8.78 through 8.83, refer to Tables IV, V, and VI.)

8.77. Integrate the appropriate chi-square density to find the probability that the variance of a random sample of size 5 from a normal population with $\sigma^2 = 25$ will fall between 20 and 30.

8.78. The claim that the variance of a normal population is $\sigma^2 = 25$ is to be rejected if the variance of a random sample of size 16 exceeds 54.668 or is less than 12.102. What is the probability that this claim will be rejected even though $\sigma^2 = 25$?

8.79. The claim that the variance of a normal population is $\sigma^2 = 4$ is to be rejected if the variance of a random sample of size 9 exceeds 7.7535. What is the probability that this claim will be rejected even though $\sigma^2 = 4$?

8.80. A random sample of size $n = 25$ from a normal population has the mean $\bar{x} = 47$ and the standard deviation $s = 7$. If we base our decision on the statistic of Theorem 8.13, can we say that the given information supports the conjecture that the mean of the population is $\mu = 42$?

8.81. A random sample of size $n = 12$ from a normal population has the mean $\bar{x} = 27.8$ and the variance $s^2 = 3.24$. If we base our decision on the statistic of Theorem 8.13, can we say that the given information supports the claim that the mean of the population is $\mu = 28.5$?

8.82. If S_1 and S_2 are the standard deviations of independent random samples of sizes $n_1 = 61$ and $n_2 = 31$ from normal populations with $\sigma_1^2 = 12$ and $\sigma_2^2 = 18$, find $P(S_1^2/S_2^2 > 1.16)$.

8.83. If S_1^2 and S_2^2 are the variances of independent random samples of sizes $n_1 = 10$ and $n_2 = 15$ from normal populations with equal variances, find $P(S_1^2/S_2^2 < 4.03)$.

8.84. Use a computer program to verify the five entries in Table IV corresponding to 11 degrees of freedom.

8.85. Use a computer program to verify the eight entries in Table V corresponding to 21 degrees of freedom.

8.86. Use a computer program to verify the five values of $f_{0.05}$ in Table VI corresponding to 7 and 6 to 10 degrees of freedom.

8.87. Use a computer program to verify the six values of $f_{0.01}$ in Table VI corresponding to $\nu_1 = 15$ and $\nu_2 = 12, 13, \ldots, 17$.

SEC. 8.7

8.88. Find the probability that in a random sample of size $n = 4$ from the continuous uniform population of Exercise 8.46, the smallest value will be at least 0.20.

8.89. Find the probability that in a random sample of size $n = 3$ from the beta population of Exercise 8.77, the largest value will be less than 0.90.