- Special probability distributions - distinguishing

Very rough notes!

- **Uniform discrete distribution:** This describes the probability distribution of any finite RV whose possible values all have the same probability.
- **Bernoulli distribution:** This describes the probability distribution of the RV X = the number of successes in one Bernoulli trial. A **Bernoulli trial** is a random process that has only two possible outcomes. Examples we have seen include flipping a coin, choosing a binary digit, answering a true-false question, answering a multiple-choice question... The Bernoulli distribution is only used as a tool to build up the binomial distribution.
- **Binomial distribution:**
  - The random variable X represents the number of successes in n independent repetitions of a Bernoulli trial, where the probability of success on any one trial is θ.
  - This distribution also represents sampling **with** replacement (which is rare), or sampling from an **infinite population**. Then X represents the number of successes in the sample. Sometimes the sample is being taken from a population which is so big that for all practical purposes it might as well be infinite: see below.
- **Negative binomial, or Pascal distribution:** This X represents the waiting time (number of Bernoulli trials) needed until a certain fixed number of binomial successes occur.

  Clue: If you are waiting until the first, or the second, or the... nth Bernoulli success, this is probably the distribution you want to use.
- **Geometric distribution:** This is the special case of the Pascal distribution for waiting time until the **first** binomial success. Think of the St. Petersburg paradox problem. But be careful that it's not waiting time for a Poisson success: see below.
- **Hypergeometric distribution:**
  - This represents sampling without replacement (i.e. normal sampling) from a **finite** population, and X represents the number of successes in the sample.
  - When the population size is very large compared to the size of the sample, this can be estimated by the binomial distribution. This is the case I mentioned above: the population size is for all practical purposes infinite.

  Clue: If you are told the size of the population the sample is being chosen from (or you already know it, as in dealing a hand of poker), then almost certainly the hypergeometric distribution is involved. But you may be able to estimate it by a binomial probability: see above.
- **Poisson distribution:**
  - This represents the probability distribution of a Poisson process: that is a random process which fulfills the three conditions given in the textbook. A Poisson process is similar to a binomial process except that the successes occur at random times (not just at iterations of some kind of trial.) Examples of Poisson processes are : radioactive decay, calls coming in to a switchboard, trucks arriving at a depot,...
  - The Poisson distribution also estimates the binomial distribution when n is large and θ is small.

  Clue: The parameter λ in the Poisson distribution represents an average number of successes per unit of "time". If you are given information of this nature, it is possible the Poisson distribution is the one you need. But it could also be the waiting time: see below.

- **Exponential distribution:**
  - This represents the probability distribution for exponential processes. You will be told if the process is exponential.
  - This X also represents the waiting time distribution for a Poisson process. That is, X represents the time gap before the first Poisson success, and also X represents the time gap between successive Poisson successes. So (loosely speaking) you have a process in which successes occur at random (continuous) times, and you are asking for the probability of having to wait a certain amount of time before the next success.