

A fair coin is tossed three times. Let X be the number of heads in the tosses minus the number of tails. Find $p_X(k)$ (otherwise known as $P(X = k)$, the probability distribution function for X .)

For this experiment we have the sample space

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

The possible values of X are 3, 1, -1, -3

The probability distribution function is:

$$p_X(3) = P(X = 3) = P(HHH) = \frac{1}{8}$$

$$p_X(1) = P(X = 1) = P(HHT) + P(HTH) + P(THH) = \frac{3}{8}$$

$$p_X(-1) = P(X = -1) = P(HTT) + P(THT) + P(TTH) = \frac{3}{8}$$

$$p_X(-3) = P(X = -3) = P(TTT) = \frac{1}{8}$$
