

Counting Methods

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- ▶ and **the number of choices at each step is the same no matter what choice was made at any previous step:**
- ▶ then the total number of outcomes is the product of the number of choices at each step.

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- ▶ then we are counting permutations.
- ▶ The number of permutations of k items chosen from a set of n items is ${}_n P_k = n(n-1) \cdots (n-k+1) = \frac{n!}{(n-k)!}$

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- ▶ We are counting outcomes to an experiment or process, where
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- ▶ we are selecting k items from a fixed set of n items
- ▶ we are selecting without replacement (each item can only be selected once)
- ▶ and the ordering of the selected items does not matter: rearranging does not give a different outcome
- ▶ then we are counting combinations.
- ▶ The number of combinations of k items chosen from a set of n items is ${}_n C_k$ or (binomial coefficient notation $\binom{n}{k} = \frac{n!}{k!(n-k)!}$)