

- b. The event “Neither appears” is the complement of the event “At least one appears.” But  $P(\text{At least one appears}) = P(L \cup D)$ . From Theorems 2.3.1 and 2.3.6, then,

$$\begin{aligned} P(\text{Neither appears}) &= 1 - P(L \cup D) \\ &= 1 - [P(L) + P(D) - P(L \cap D)] \\ &= 1 - [0.40 + 0.30 - 0.05] \\ &= 0.35 \end{aligned}$$

**Example**  
**2.3.5**

Having endured (and survived) the mental trauma that comes from taking two years of chemistry, a year of physics, and a year of biology, Biff decides to test the medical school waters and sends his MCATs to two colleges,  $X$  and  $Y$ . Based on how his friends have fared, he estimates that his probability of being accepted at  $X$  is 0.7, and at  $Y$  is 0.4. He also suspects there is a 75% chance that at least one of his applications will be rejected. What is the probability that he gets at least one acceptance?

Let  $A$  be the event “School  $X$  accepts him” and  $B$  the event “School  $Y$  accepts him.” We are given that  $P(A) = 0.7$ ,  $P(B) = 0.4$ , and  $P(A^C \cup B^C) = 0.75$ . The question is asking for  $P(A \cup B)$ .

From Theorem 2.3.6,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Recall from Question 2.2.32 that  $A^C \cup B^C = (A \cap B)^C$ , so

$$P(A \cap B) = 1 - P[(A \cap B)^C] = 1 - 0.75 = 0.25$$

It follows that Biff’s prospects are not all that bleak—he has an 85% chance of getting in somewhere:

$$\begin{aligned} P(A \cup B) &= 0.7 + 0.4 - 0.25 \\ &= 0.85 \end{aligned}$$

**Comment** Notice that  $P(A \cup B)$  varies directly with  $P(A^C \cup B^C)$ :

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - [1 - P(A^C \cup B^C)] \\ &= P(A) + P(B) - 1 + P(A^C \cup B^C) \end{aligned}$$

If  $P(A)$  and  $P(B)$ , then, are fixed, we get the curious result that Biff’s chances of getting at least one acceptance increase if his chances of at least one rejection increase.

## Questions

**2.3.1.** According to a family-oriented lobbying group, there is too much crude language and violence on television. Forty-two percent of the programs they screened had language they found offensive, 27% were too violent, and 10% were considered excessive in both language and violence. What percentage of programs did comply with the group’s standards?

**2.3.2.** Let  $A$  and  $B$  be any two events defined on  $S$ . Suppose that  $P(A) = 0.4$ ,  $P(B) = 0.5$ , and  $P(A \cap B) = 0.1$ . What is the probability that  $A$  or  $B$  but not both occur?

**2.3.3.** Express the following probabilities in terms of  $P(A)$ ,  $P(B)$ , and  $P(A \cap B)$ .

- (a)  $P(A^c \cup B^c)$   
 (b)  $P(A^c \cap (A \cup B))$

**2.3.4.** Let  $A$  and  $B$  be two events defined on  $S$ . If the probability that at least one of them occurs is 0.3 and the probability that  $A$  occurs but  $B$  does not occur is 0.1, what is  $P(B)$ ?

**2.3.5.** Suppose that three fair dice are tossed. Let  $A_i$  be the event that a 6 shows on the  $i$ th die,  $i = 1, 2, 3$ . Does  $P(A_1 \cup A_2 \cup A_3) = \frac{1}{2}$ ? Explain.

**2.3.6.** Events  $A$  and  $B$  are defined on a sample space  $S$  such that  $P((A \cup B)^c) = 0.5$  and  $P(A \cap B) = 0.2$ . What is the probability that either  $A$  or  $B$  but not both will occur?

**2.3.7.** Let  $A_1, A_2, \dots, A_n$  be a series of events for which  $A_i \cap A_j = \emptyset$  if  $i \neq j$  and  $A_1 \cup A_2 \cup \dots \cup A_n = S$ . Let  $B$  be any event defined on  $S$ . Express  $B$  as a union of intersections.

**2.3.8.** Draw the Venn diagrams that would correspond to the equations (a)  $P(A \cap B) = P(B)$  and (b)  $P(A \cup B) = P(B)$ .

**2.3.9.** In the game of “odd man out” each player tosses a fair coin. If all the coins turn up the same except for one, the player tossing the different coin is declared the odd man out and is eliminated from the contest. Suppose that three people are playing. What is the probability that someone will be eliminated on the first toss? (*Hint:* Use Theorem 2.3.1.)

**2.3.10.** An urn contains twenty-four chips, numbered 1 through 24. One is drawn at random. Let  $A$  be the event that the number is divisible by 2 and let  $B$  be the event that the number is divisible by 3. Find  $P(A \cup B)$ .

**2.3.11.** If State’s football team has a 10% chance of winning Saturday’s game, a 30% chance of winning two weeks from now, and a 65% chance of losing both games, what are their chances of winning exactly once?

**2.3.12.** Events  $A_1$  and  $A_2$  are such that  $A_1 \cup A_2 = S$  and  $A_1 \cap A_2 = \emptyset$ . Find  $p_2$  if  $P(A_1) = p_1$ ,  $P(A_2) = p_2$ , and  $3p_1 - p_2 = \frac{1}{2}$ .

**2.3.13.** Consolidated Industries has come under considerable pressure to eliminate its seemingly discriminatory

hiring practices. Company officials have agreed that during the next five years, 60% of their new employees will be females and 30% will be minorities. One out of four new employees, though, will be a white male. What percentage of their new hires will be minority females?

**2.3.14.** Three events— $A$ ,  $B$ , and  $C$ —are defined on a sample space,  $S$ . Given that  $P(A) = 0.2$ ,  $P(B) = 0.1$ , and  $P(C) = 0.3$ , what is the smallest possible value for  $P[(A \cup B \cup C)^c]$ ?

**2.3.15.** A coin is to be tossed four times. Define events  $X$  and  $Y$  such that

- $X$ : first and last coins have opposite faces  
 $Y$ : exactly two heads appear

Assume that each of the sixteen head/tail sequences has the same probability. Evaluate

- (a)  $P(X^c \cap Y)$   
 (b)  $P(X \cap Y^c)$

**2.3.16.** Two dice are tossed. Assume that each possible outcome has a  $\frac{1}{36}$  probability. Let  $A$  be the event that the sum of the faces showing is 6, and let  $B$  be the event that the face showing on one die is twice the face showing on the other. Calculate  $P(A \cap B^c)$ .

**2.3.17.** Let  $A$ ,  $B$ , and  $C$  be three events defined on a sample space,  $S$ . Arrange the probabilities of the following events from smallest to largest:

- (a)  $A \cup B$   
 (b)  $A \cap B$   
 (c)  $A$   
 (d)  $S$   
 (e)  $(A \cap B) \cup (A \cap C)$

**2.3.18.** Lucy is currently running two dot-com scams out of a bogus chatroom. She estimates that the chances of the first one leading to her arrest are one in ten; the “risk” associated with the second is more on the order of one in thirty. She considers the likelihood that she gets busted for both to be 0.0025. What are Lucy’s chances of avoiding incarceration?

## 2.4 Conditional Probability

In Section 2.3, we calculated probabilities of certain events by manipulating other probabilities whose values we were given. Knowing  $P(A)$ ,  $P(B)$ , and  $P(A \cap B)$ , for example, allows us to calculate  $P(A \cup B)$  (recall Theorem 2.3.6). For many real-world situations, though, the “given” in a probability problem goes beyond simply knowing a set of other probabilities. Sometimes, we know *for a fact* that certain events *have already occurred*, and those occurrences may have a bearing on the