2.4.2) Use the definitions: $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and $P(B|A) = \frac{P(A \cap B)}{P(A)}$, and then do some algebra to find that $P(A \cap B) = \frac{1}{10}$

2.4.12) The probability of getting at least two heads, given that there are at most two heads, is $\frac{P(\text{2 heads})}{P(\text{0,1, or 2 heads})}$
$P(2 \text{ heads}) = \frac{3}{8}$
$P(0, 1, \text{ or } 2 \text{ heads}) = 1 - P(3 \text{ heads})$, so
$P(0, 1, \text{ or } 2 \text{ heads}) = 1 - \frac{1}{8} = \frac{7}{8}$

So $\frac{P(\text{2 heads})}{P(\text{0,1, or 2 heads})} = \frac{1/8}{7/8} = \frac{1}{7}$

2.4.14) We want to find the probability that all four cards are aces, given that at least three are aces. Assuming that the cards are dealt at random (i.e. the deck is well-shuffled), all four-card hands are equally likely and we are told that the probability of any specific four-card hand is $\frac{1}{270725}$. (We will learn later in the course how to get this number.)
There is only one hand that has four aces, so its probability is $\frac{1}{270725}$.
Now we count the number of hands that have at least 3 aces: There are four ways to get three aces, namely, (hearts diamonds clubs), (hearts diamonds spades), (hearts clubs spades), and (diamonds clubs spades). For each of those four sets of three aces, there are 48 non-ace cards that could be the fourth card in the hand. So the total number of four-card hands with three aces is $4(48) = 192$. Thus the total number of four-card hands with at least 3 aces is $192+1 = 193$, and the probability of getting at least 3 aces is $\frac{193}{270725}$.

Finally, the probability of getting four aces given that the hand contains at least 3 aces is: $\frac{1/270725}{193/270725} = \frac{1}{193}$

2.4.25) Set up notation:
Let A_1 = "The ball bearings came from supplier 1", similarly for A_2, A_3
Let D = "The ball bearings are defective"

From the information in the problem,
$P(A_1 = 0.5$, $P(A_2 = 0.3$, $P(A_3 = 0.2$
$P(D|A_1 = 0.02$, $P(D|A_2 = 0.03$, $P(D|A_3 = 0.04$

Then $P(D) = (0.5)(0.02) + (0.3)(0.03) + (0.2)(0.04) = 0.027$

2.7% of the ball bearings are defective.

2.4.26) At the first step, $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

Let the event 6 = "The die shows 6 or the sum on the two dice is 6"
Then at the second step, $P(6|H) = \frac{1}{6}$, $P(6|T) = \frac{5}{36}$ because there are 5 ways to get a sum of 6, namely, (1,5), (2,4), (3,3), (4,2), (5,1).

So $P(6) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{36}\right) = \frac{11}{72}$

2.4.31) Let H = "The person has HIV", T = "The person tests positive"

From the information in the problem,
$P(H) = 0.0001$, $P(H') = 1 - 0.0001 = 0.9999$
$P(T|H) = 0.999$, $P(T'|H') = 0.9999$

To find $P(T)$ we need to know $P(T|H')$ which is $1 - P(T'|H') = 0.0001$

So $P(T) = P(H)P(T|H) + P(T)P(T]H) = (0.0001)(0.999) + (0.0001)(0.9999) = 0.00019989$

The probability that a randomly selected person from this population tests positive is about 0.02%

Note: The important thing in these problems that involve conditional probabilities is to make sure that you understand how to recognize a conditional probability when it is described. Look especially in the last three problems above which use the "Pre-Bayes" theorem. How do we know that those conditional probabilities are conditional, from the way they are given in the problem? This is important because conditional probabilities arise in many real life situations and they will be described in a number of ways, rarely if ever using the word "given"! People just don't talk that way in real life!