Probability - Important Formulas and Rules

Addition Rules:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
special case: If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$

Probability of a complement:
$P(A) + P(A') = 1$
This is because A and A' are m.e. and their union is the whole sample space S.
This gives the useful formula
$P(A') = 1 - P(A)$
Also useful: for conditional probabilities, $P(A|B) + P(A'|B) = 1$; $P(A \cap B) + P(A' \cap B) = P(B)$

Probability of the empty set:
$P(\emptyset) = 0$
(Because the empty set is the complement of the sample space)

Probability of a subset:
If $A \subset B$, then $P(A) \leq P(B)$

DeMorgan's Laws:
$(A \cap B)' = A' \cup B'$ so $P(A \cap B) = 1 - P(A' \cup B')$
$(A \cup B)' = A' \cap B'$ so $P(A \cup B) = 1 - P(A' \cap B')$

Equally likely outcomes sample spaces:
If S is a finite sample space consisting of n outcomes, they are said to be equally likely outcomes if the probability of each outcome in S is the same: each outcome must have probability $\frac{1}{n}$.
Then $P(A) = \frac{|A|}{|S|} = \frac{\text{the number of outcomes in A}}{\text{the total number of possible outcomes}}$

Or colloquially, $P(A) = \frac{\text{the number of favorable outcomes}}{\text{the total number of outcomes}}$
where "favorable" just means that they are outcomes we are looking for (they are in the event A), not necessarily that they are good or desirable.

Conditional probabilities:
$P(A|B) = \frac{P(A \cap B)}{P(B)}$
Note: $P(A|B)$ is also written $P_B(A)$, and is read as "the probability of A given B".
In the case of an equally likely outcomes sample space, we can compute it by $P(A|B) = \frac{|A \cap B|}{|B|}$

Independent events:
The events A and B are **independent** if $P(A|B) = P(A)$ or $P(B|A) = P(B)$
Note: if either one of these is true, they are both true and A and B are independent.

Multiplication rules:
$P(A \cap B) = P(A|B)P(B)$
also $P(A \cap B) = P(B|A)P(A)$
special case: A and B are independent if and only if $P(A \cap B) = P(A)P(B)$
(This gives another way to check for independence.)

Bayes' Theorem:
If $B_1, B_2, \ldots, B_n$ are pairwise mutually exclusive events whose union is S, and A is another event in S, then

$$P(B_j|A) = \frac{P(A|B_j}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

We say that such events $B_1, B_2, \ldots, B_n$ are **mutually exclusive and exhaustive**, or that they are a **partition of the sample space**, or (sometimes) that they **tile the sample space**.
This theorem provides a way to compute the prior probability $P(B_j|A)$ if the posterior probabilities $P(A|B_i)$ are all known, as well as the probabilities $P(B_i)$, for all $i = 1, 2, \ldots, n$