

Testing strategy for series including power series

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- ▶ If the limit of the terms is not 0, the series will diverge.

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- ▶ A geometric series $\sum_{n=0}^{\infty} r^n$ will converge if $|r| < 1$ and otherwise diverge.

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- ▶ Limit comparison may work if the direct comparison inequality is in the wrong direction.

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- ▶ If the terms $a_n = f(n)$ for some positive, decreasing function $f(x)$, then the Integral Test may work.