

Consider the series $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$

Evaluate the limit

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\begin{aligned} \left| \frac{a_{n+1}}{a_n} \right| &= \frac{\left(\frac{10^{n+1}}{(n+1+1)4^{2(n+1)+1}} \right)}{\left(\frac{10^n}{(n+1)4^{2n+1}} \right)} \\ &= \left(\frac{10^{n+1}}{(n+2)4^{2n+3}} \right) \left(\frac{10^n}{(n+1)4^{2n+1}} \right) \\ &= \frac{10(n+1)}{(n+2)4^2} = \frac{5(n+1)}{8(n+2)} \end{aligned}$$

$$\text{So } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{5(n+1)}{8(n+2)} = \frac{5}{8}$$

Since $\frac{5}{8} < 1$, the series converges absolutely (and so also converges conditionally), by the Ratio Test.
