For more information and answers or partial solutions, see the class homework blog at https://openlab.citytech.cuny.edu/shavermat1575fall2017/

Topics for this test: Please note that you also need to know the methods covered in the previous tests!

- Taylor polynomials and using them to estimate values of a function.
- Sequences, and the theorems about convergence:
  - A convergent sequence is bounded (so an unbounded sequence cannot converge).
  - If the sequence of absolute values $\{|a_n|\}$ converges, then the original sequence $\{a_n\}$ converges.
  - A bounded monotonic sequence converges.
- Series, partial sums, and the theorems about convergence/divergence (so far)
  - Integral comparison
  - (Direct) comparison
  - Limit comparison
- Special series: geometric series and p-series; telescoping series

Instructions: These problems are for you to use to test yourself, **after** you have practiced with the routine homework assignments, to see how ready you are for Test 3. They are not meant as a substitute for regular and diligent practice!

Do the following problems as if you were taking a test: without notes or textbook, and give yourself a time limit as stated at the start of each self-test. At the end of that time, check your answers against the answers posted on the blog. Then review as needed from the relevant sections, before you re-test.

Self-Test 1: allow 60 minutes.

1) Find the 6th degree Taylor polynomial centered at $\frac{\pi}{2}$ for $f(x) = \sin(x)$, and use it to find an estimate for $\sin\left(\frac{2\pi}{3}\right)$ to nine decimal places. How close is your estimate to the true value?

2) Determine whether each of the following sequences converge or diverge: you must explain how you reached your conclusion.

   a) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$
   b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$
   c) $\{a_n\} = \{\frac{n}{n^2-1}\}$ for n starting at 2
   d) $\{a_n\} = \{\frac{n+1}{n}\}$

3) For each of the following series, find the partial sums S_1, S_2, S_3, S_4, and determine whether the series converges or diverges. You must explain how you reached your conclusion. If the series converges, give the sum if you can.

   a) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots$
   b) $5 + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} + 1 + \frac{5}{6} + \ldots$
   c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$
   d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$ (Hint: this converges, but it's rather tricky to find the value of the sum.)

Self-Test 2: allow 60 minutes.

1) Find the 3rd degree Taylor polynomial centered at 0 (the Maclauren polynomial) for $f(x) = \sqrt[3]{x-1}$, and use it to find an estimate for $\sqrt[3]{-0.5}$ to four decimal places. How close is your estimate to the true value?

2) Determine whether each of the following sequences converge or diverge: you must explain how you reached your conclusion.

   a) $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \ldots$

   b) $1, 1, 1, 1, 1, 1, \ldots$

   c) $\{a_n\} = \{\frac{n+1}{n!}\}$

   d) $\{a_n\} = \{\frac{|\sin(n)|}{n}\}$

3) For each of the following series, find the partial sums S_1, S_2, S_3, S_4, and determine whether the series converges or diverges. You must explain how you reached your conclusion. If the series converges, give the sum if you can.

   a) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \ldots$

   b) $5 - \frac{5}{2} + \frac{5}{4} - \frac{5}{8} + \frac{5}{16} + \ldots$

   c) $\displaystyle\sum_{n=4}^{\infty} \frac{n}{n^2 - 2n - 3}$

   d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$