

Series: Ratio and Root tests for convergence

Sybil Shaver

November 13, 2017

Ratio Test for convergence

For the series $\sum a_n$

▶ define $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$

Ratio Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).

Ratio Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).
- ▶ If $L > 1$ the series is divergent.

Ratio Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).
- ▶ If $L > 1$ the series is divergent.
- ▶ If $L = 1$ the series may be conditionally convergent, absolutely convergent, or divergent. The test is inconclusive.

Root Test for convergence

For the series $\sum a_n$

▶ define $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$

Root Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).

Root Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).
- ▶ If $L > 1$ the series is divergent.

Root Test for convergence

For the series $\sum a_n$

- ▶ define $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$
- ▶ If $L < 1$ the series is absolutely convergent (and so it is convergent).
- ▶ If $L > 1$ the series is divergent.
- ▶ If $L = 1$ the series may be conditionally convergent, absolutely convergent, or divergent. The test is inconclusive.