Part A: $\int 5\tan^2(x)\mathrm{d}x$

We use the identity $\tan^2(x)+1=\sec^2(x) \implies \tan^2(x)=\sec^2(x)-1$

$$\begin{aligned}\int 5\tan^2(x)\mathrm{d}x &= 5\int \left(\sec^2(x)-1\right)\mathrm{d}x\\ &= 5\left(\tan(x)-x\right)+C\\ &= 5\tan(x)-5x+C\end{aligned}$$

Part B: $\displaystyle\int_0^9 \frac{\mathrm{d}x}{\sqrt{100+x^2}}$

We use the substitution $x=10\tan(\theta) \implies \mathrm{d}x=10\sec^2(\theta)\mathrm{d}\theta$
Also change the bounds of integration:
$x=0 \implies \theta=\tan^{-1}\left(\frac{0}{10}\right)=0$
$x=9 \implies \theta=\tan^{-1}\left(\frac{9}{10}\right)$ which we will have to work with later.

$$\begin{aligned}\int_0^9 \frac{\mathrm{d}x}{\sqrt{100+x^2}} &= \int_0^{\tan^{-1}(9/10)} \frac{10\sec^2(\theta)\mathrm{d}\theta}{\sqrt{10^2+10^2\tan^2(\theta)}}\\ &= \int_0^{\tan^{-1}(9/10)} \frac{10\sec^2(\theta)\mathrm{d}\theta}{10\sqrt{1+\tan^2(\theta)}}\\ &= \int_0^{\tan^{-1}(9/10)} \frac{\sec^2(\theta)\mathrm{d}\theta}{\sqrt{\sec^2(\theta)}}\\ &= \int_0^{\tan^{-1}(9/10)} \frac{\sec^2(\theta)\mathrm{d}\theta}{\sec(\theta)}\\ &= \int_0^{\tan^{-1}(9/10)} \sec(\theta)\mathrm{d}\theta\\ &= \ln\left|\sec(\theta)+\tan(\theta)\right|\Big|_0^{\tan^{-1}(9/10)}\end{aligned}$$

In order to do the substitution, we want to know what $\sec(\theta)$ is if $\tan(\theta)=\frac{9}{10}$. If you draw a right triangle with the side opposite θ being 9 and the side adjacent to θ being 10, you can find the hypotenuse is $\sqrt{181}$, so $\sec(\theta)=\frac{\sqrt{181}}{10}$.

$$\begin{aligned}\ln\left|\sec(\theta)+\tan(\theta)\right|\Big|_0^{\tan^{-1}(9/10)} &= \ln\left|\frac{\sqrt{181}}{10}+\frac{9}{10}\right| - \ln\left|\sec(0)+\tan(0)\right|\\ &= \ln\left|\frac{9+\sqrt{181}}{10}\right| - \ln|1+0|\\ &= \ln\left|\frac{9+\sqrt{181}}{10}\right|\end{aligned}$$

Note: We don't need the absolute value signs in that expression because $\frac{9+\sqrt{181}}{10}$ is positive.