Part A: Find the Taylor polynomial of degree 6 for $f(x) = \ln(x)$ centered at 1: show all your work here. (You must derive it, not just write it down from memory.)

First find all the derivatives through the sixth derivative and evaluate $\ln(x)$ and its first six derivatives at $x = 1$:

$f(x) = \ln(x)$	$f(1) = \ln(1) = 0$
$f'(x) = \frac{1}{x} = x^{-1}$	$f'(1) = 1^{-1} = 1$
$f''(x) = -x^{-2}$	$f'''(1) = -(1^{-2}) = -1$
$f'''(x) = 2x^{-3}$	$f'''(1) = 2$
$f^{(4)}(x) = -6x^{-4}$	$f^{(4)}(1) = -6$
$f^{(5)}(x) = 24x^{-5}$	$f^{(5)}(1) = 24$
$f^{(6)}(x) = -120x^{-6}$	$f^{(6)}(1) = -120$

Now substitute these into the formula for the 6th degree Taylor polynomial

$$
\begin{aligned}
T_6 &= f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 + \frac{f^{(5)}(1)}{5!}(x-1)^5 + \frac{f^{(6)}(1)}{6!}(x-1)^6 \\
&= 0 + 1(x-1) + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4 + \frac{24}{5!}(x-1)^5 + \frac{-120}{6!}(x-1)^6 \\
&= (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 - \frac{1}{6}(x-1)^6
\end{aligned}
$$

Part B: Does each of the following converge or diverge? If convergent, give the limit. If it diverges, explain how you know that.

1) $\{a_n\} = \{\frac{2^n-1}{2^n}\}$

This can be rewritten:

$$\frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n} \to 1 - 0 = 1 \text{ as } n \to \infty$$

So the sequence converges to 1.

2) $\{a_n\} = \{(-1)^{n+1}\}$

This sequence alternates between 1 and -1, so it diverges.