

Note: remember you can always check the integrals by differentiating.

1) $\int 5x\sqrt{x^2+3}dx$: use substitution with $u = x^2 + 3$, so $du = 2xdx$

$$\begin{aligned}\int 5x\sqrt{x^2+3}dx &= \frac{5}{2} \int \sqrt{u}du \\ &= \frac{5}{2} \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{5}{3} (x^2+3)^{\frac{3}{2}} + C\end{aligned}$$

2) $\int x^2 \sin(x)dx$; integrate by parts (twice)

First integration by parts let $u = x^2 \implies du = 2xdx$; $dv = \sin(x)dx \implies v = -\cos(x)$

$$\int x^2 \sin(x)dx = -x^2 \cos(x) + 2 \int x \cos(x)dx$$

Now integrate by parts again with $u = x \implies du = dx$; $dv = \cos(x)dx \implies v = \sin(x)$

$$-x^2 \cos(x) + 2 \int x \cos(x)dx = -x^2 \cos(x) + 2x \sin(x) - 2 \int \sin(x)dx$$

$$= -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C$$

$$\text{or } (2 - x^2) \cos(x) + 2x \sin(x) + C$$
