

**Please do not write in the margins of the page!**

**Part A:** For the series  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$

- 1) Write a formula for the general term  $a_n$  (not the partial sum) of this series.

$$a_n = \frac{1}{3^n}$$

- 2) Compute the partial sums  $S_2$ ,  $S_3$ , and  $S_4$

$$\begin{aligned} S_2 &= \frac{1}{3} + \frac{1}{9} = \frac{4}{9} \\ S_3 &= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} = \frac{13}{27} \\ S_4 &= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} = \frac{40}{81} \end{aligned}$$

- 3) Is this series a geometric series, a p-series, or neither?

This is a geometric series ( $r = \frac{1}{3}$ )

**Part B:** For the series  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$

- 1) Write a formula for the general term  $a_n$  (not the partial sum) of this series.

$$a_n = \frac{1}{n+1}$$

- 2) Compute the partial sums  $S_2$ ,  $S_3$ , and  $S_4$

$$\begin{aligned} S_2 &= \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \\ S_3 &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12} \\ S_4 &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \end{aligned}$$

- 3) Is this series a geometric series, a p-series, or neither?

This is a p-series ( $p=1$ ): this series is also called the harmonic series.

**Part C:** For the telescoping series  $\sum_{n=1}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n+2} \right)$

- 1) Compute the partial sums  $S_2$ ,  $S_3$ , and  $S_4$

$$\begin{aligned} S_2 &= \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) = \frac{1}{4} \\ S_3 &= \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \left( \frac{1}{4} - \frac{1}{5} \right) = \frac{1}{2} - \frac{1}{5} = \frac{3}{10} \\ S_4 &= \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \left( \frac{1}{4} - \frac{1}{5} \right) + \left( \frac{1}{5} - \frac{1}{6} \right) = \frac{1}{2} - \frac{1}{6} = \frac{1}{3} \end{aligned}$$

- 2) Find the sum of the infinite series.

$$\text{The sum is } \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \left( \frac{1}{4} - \frac{1}{5} \right) + \left( \frac{1}{5} - \frac{1}{6} \right) + \dots = \frac{1}{2}$$