Part A: Find the integral and simplify: show **all** your work.

$$\int \frac{1}{(x+2)\left(x^2+1\right)} \mathrm{d}x$$

Find the partial fractions decomposition:

$\frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$1 = A(x^2+1) + (Bx+C)(x+2)$

$1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$1 = (A+B)x^2 + (2B+C)x + (A+2C)$

$\implies A+B=0,$

$2B+C=0,$

$A+2C=1$

Solve that system to find that $A=\frac{1}{5}$, $B=-\frac{1}{5}$, and $C=\frac{2}{5}$

So now we have

$$\begin{aligned}\int \frac{1}{(x+2)\left(x^2+1\right)} \mathrm{d}x &= \frac{1}{5}\int \frac{1}{x+2}\mathrm{d}x - \frac{1}{5}\int \frac{x}{x^2+1}\mathrm{d}x + \frac{2}{5}\int \frac{1}{x^2+1}\mathrm{d}x \\ &= \frac{1}{5}\ln|x+2| - \frac{1}{10}\ln\left(x^2+1\right) + \frac{2}{5}\tan^{-1}(x) + C\end{aligned}$$

(You can have good practice filling in the details of the three integrations in that last step!)

Part B: Find the value of the improper integral if it exists, otherwise show that it diverges: show all your work and simplify your answer.

$$\begin{aligned}\int_9^\infty \frac{\mathrm{d}x}{x^{3/2}} &= \lim_{b\to\infty} \int x^{-3/2}\mathrm{d}x \\ &= \lim_{b\to\infty}\left[-2x^{-1/2}\right]_9^b \\ &= -2\left(9^{-1/2}\right) - \lim_{b\to\infty}\left(-2b^{-1/2}\right) \\ &= -\frac{2}{3} + 2\lim_{b\to\infty}\left(\frac{1}{\sqrt{b}}\right) \\ &= -\frac{2}{3} + 2(0) = -\frac{2}{3}\end{aligned}$$