Part A: Find the improper integral and simplify: show **all** your work.

$\int_0^5 \frac{1}{\sqrt{x}}\mathrm{d}x$

$$\begin{aligned}\int_0^5 \frac{1}{\sqrt{x}}\mathrm{d}x &= \lim_{a\to 0^+}\int_a^5 \frac{1}{\sqrt{x}}\mathrm{d}x\\ &= \lim_{a\to 0^+}\left[2\sqrt{x}\right]_a^5\\ &= \lim_{a\to 0^+}\left[2\sqrt{5}-2\sqrt{a}\right]\\ &= 2\sqrt{5}-0 = 2\sqrt{5}\end{aligned}$$

Part B: Use a comparison test to show whether the improper integral converges or diverges: state clearly what function you are using for comparison

1) $\int_0^\infty \frac{x}{x^2+\cos(x)}\mathrm{d}x$

Let $f(x) = \frac{x}{x^2+\cos(x)}$ and we can use the Limit Comparison Theorem with the function $g(x) = \frac{1}{x}$:

$\frac{f(x)}{g(x)} = \frac{1}{x}\cdot\frac{x^2+\cos(x)}{x} = \frac{x^2+\cos(x)}{x^2}$

$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{x^2+\cos(x)}{x^2} = 1 > 0$

So since $\int_1^\infty \frac{1}{x}\mathrm{d}x$ diverges, so does the original integral.

2) $\int_1^\infty e^{-x}\ln(x)\mathrm{d}x$

Since $\ln(x) < x$ for all x in $(1,\infty)$, we can use comparison with the integral of xe^{-x}, which converges: (integrate by parts)

$$\begin{aligned}\int_1^\infty xe^{-x}\mathrm{d}x &= \lim_{b\to\infty}\int_1^b xe^{-x}\ln(x)\mathrm{d}x\\ &= \lim_{b\to\infty}\left[-xe^{-x}\right]_1^b + \lim_{b\to\infty}\int_1^b \left(e^{-x}\right)\mathrm{d}x\\ &= \lim_{b\to\infty}\left[-be^{-b}+e^{-1}\right] + \lim_{b\to\infty}\left[-e^{-x}\right]_1^b\\ &= 0+e^{-1}-0+e^{-1} = 2e^{-1}\end{aligned}$$

So since this integral converges, so does the original integral.