1) Find $g'(x)$ if $g(x) = \int_{-1}^{x} \sqrt{t^2+1}\,\mathrm{d}t$

$g'(x) = \sqrt{x^2+1}$ by the FTC part II

2) For the function $g(x)$ given in problem 1, find $g''(x)$

$$g''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+1}\right)$$

$$= \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+1}}$$

3) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \int_{x}^{x^2} \sqrt{t}\sin t^2 \mathrm{d}t$

First rewrite the integral so we can use FTC (and the Chain Rule)

$$\begin{aligned} y &= \int_{x}^{x^2} \sqrt{t}\sin t^2 \mathrm{d}t \\ &= \int_{x}^{0} \sqrt{t}\sin t^2 \mathrm{d}t + \int_{0}^{x^2} \sqrt{t}\sin t^2 \mathrm{d}t \\ &= -\int_{0}^{x} \sqrt{t}\sin t^2 \mathrm{d}t + \int_{0}^{x^2} \sqrt{t}\sin t^2 \mathrm{d}t \end{aligned}$$

Now use FTC part II (and the Chain Rule, for the second integral):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{x}\sin x^2 + \sqrt{x^2}\sin\left(x^2\right)^2 \cdot 2x$$

$$= -\sqrt{x}\sin x^2 + 2x|x|\sin x^4$$