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- The area is defined as a limit of Riemann sums, which are sums of areas of rectangles.

Some facts about definite integrals:

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- $\int_b^a (f(x) + g(x))\,\mathrm{d}x = \int_b^a f(x)\mathrm{d}x + \int_b^a g(x)\mathrm{d}x$

Example 1: finding a definite integral by computing an area

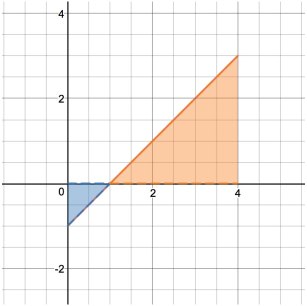
Find the definite integral $\int_0^4 (x-1)\mathrm{d}x$

- First draw the graph of $f(x) = x - 1$ between $x = 0$ and $x = 4$

Example 1: finding a definite integral by computing an area

Find the definite integral $\int_0^4 (x-1)\mathrm{d}x$

- First draw the graph of $f(x) = x - 1$ between $x = 0$ and $x = 4$
- This area consists of two triangles, one below the x-axis and one above the x-axis.



Blue area $= \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$
Orange area $= \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$

$$\int_0^4 (x-1)\mathrm{d}x = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$$

The Fundamental Theorem of Calculus (part 2)

- If $F(x)$ is any antiderivative of $f(x)$ on the interval $[a, b]$, then the integral of $f(x)$ from a to b can be computed by $\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$

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$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$$
- Note: The general antiderivative $\int f(x)\mathrm{d}x$ is also called the **indefinite integral of** $f(x)$ because of this.

Example 2

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- See the next slide for a better notation

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- Read it as: $\frac{x^3}{3}$ evaluated from 0 to 1.

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- $= e^3 - e$

Example 4

Evaluate the integral $\displaystyle\int_3^6 \frac{\mathrm{d}x}{x}$

- Note: $\int_3^6 \frac{\mathrm{d}x}{x}$ is a short way to write $\int_3^6 \left(\frac{1}{x}\right) \mathrm{d}x$

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- $= \ln(2)$