Logarithmic Differentiation

This is used to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ when y is a function which has x in both the base and the exponent.
For example,

- $y = x^x$
- $y = x^{\sin(x)}$
- $y = (1 + x)^{\frac{1}{x}}$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation: $\ln(y) = \ln\left(x^x\right)$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation:
  $\ln(y) = \ln\left(x^x\right)$
- Then use the "log of a power" rule:
  $\ln(y) = x\ln\left(x\right)$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation:
  $\ln(y) = \ln\left(x^x\right)$
- Then use the "log of a power" rule:
  $\ln(y) = x \ln\left(x\right)$
- Now differentiate implicitly:
  $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(y)) = \frac{\mathrm{d}}{\mathrm{d}x}(x \ln\left(x\right))$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation:
  $\ln(y) = \ln(x^x)$
- Then use the "log of a power" rule:
  $\ln(y) = x\ln(x)$
- Now differentiate implicitly:
  $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(y)) = \frac{\mathrm{d}}{\mathrm{d}x}(x\ln(x))$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = (1)(\ln(x)) + x\left(\frac{1}{x}\right)$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation:
  $\ln(y) = \ln\left(x^x\right)$
- Then use the "log of a power" rule:
  $\ln(y) = x\ln(x)$
- Now differentiate implicitly:
  $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(y)) = \frac{\mathrm{d}}{\mathrm{d}x}(x\ln(x))$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = (1)(\ln(x)) + x\left(\frac{1}{x}\right)$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \ln(x) + 1$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation: $\ln(y) = \ln\left(x^x\right)$
- Then use the "log of a power" rule: $\ln(y) = x\ln(x)$
- Now differentiate implicitly: $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(y)) = \frac{\mathrm{d}}{\mathrm{d}x}(x\ln(x))$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = (1)(\ln(x)) + x\left(\frac{1}{x}\right)$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \ln(x) + 1$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = y(\ln(x) + 1)$

Example 1:

Use logarithmic differentiation to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^x$.

- First take the natural logarithm of both sides of the equation: $\ln(y) = \ln\left(x^x\right)$
- Then use the "log of a power" rule: $\ln(y) = x \ln(x)$
- Now differentiate implicitly: $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(y)) = \frac{\mathrm{d}}{\mathrm{d}x}(x \ln(x))$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = (1)(\ln(x)) + x\left(\frac{1}{x}\right)$
- $\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \ln(x) + 1$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = y(\ln(x) + 1)$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(\ln(x) + 1)$