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- $f(u(x)) = (\sin(x))^2 = \sin^2(x)$

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

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The Chain Rule

For the composite function $y = f(u(x))$, the derivative is
$y' = f'(u(x)) \cdot u'(x)$

Another way to write this is
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

"The derivative of the outer times the derivative of the inner."