Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$
- If $f(x) = \sqrt{x}$ and $u(x) = x^2 + 1$, what is $f(u(x))$?

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$
- If $f(x) = \sqrt{x}$ and $u(x) = x^2 + 1$, what is $f(u(x))$?
- $f(u(x)) = \sqrt{x^2 + 1}$

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$

- If $f(x) = e^x$ and $u(x) = 3x$, what is $f(u(x))$?

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$

- If $f(x) = e^x$ and $u(x) = 3x$, what is $f(u(x))$?
- $f(u(x)) = e^{3x}$

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$
- If $f(x) = x^2$ and $u(x) = \sin(x)$, what is $f(u(x))$?

Composite Functions

- **Recall that we find the composite function $f(u(x))$ by substituting $u(x)$ in place of x in the formula for f**
- **Example:** if $f(x) = x^4$ and $u(x) = 5 - x$, then $f(u(x)) = (5 - x)^4$

- If $f(x) = x^2$ and $u(x) = \sin(x)$, what is $f(u(x))$?
- $f(u(x)) = (\sin(x))^2 = \sin^2(x)$

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.
- What are the outer and inner functions for $\tan(x^2 + 4)$?

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.
- What are the outer and inner functions for $\tan(x^2 + 4)$?
- The outer function is $f(x) = \tan(x)$
  and the inner function is $u(x) = x^2 + 4$

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.

- What are the outer and inner functions for $\cos^3(x)$?

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.

- What are the outer and inner functions for $\cos^3(x)$?
- Recall that $\cos^3(x)$ means $(\cos(x))^3$

Recognizing composite functions

In $f(u(x))$, we often refer to f as the **outer function** and u as the **inner function**.

- **Example:** for the function $\ln(\sin(x))$,
  the outer function is $f(x) = \ln(x)$
  and the inner function is $u(x) = \sin(x)$.

- What are the outer and inner functions for $\cos^3(x)$?
- Recall that $\cos^3(x)$ means $(\cos(x))^3$
- The outer function is $f(x) = x^3$
  and the inner function is $u(x) = \cos(x)$

The Chain Rule

For the composite function $y = f(u(x))$, the derivative is
$y' = f'(u(x)) \cdot u'(x)$

Another way to write this is
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

"The derivative of the outer times the derivative of the inner."

Special case: the General Power Rule

For the function $y = (u(x))^n$, the derivative is
$y' = n(u(x))^{n-1} \cdot u'(x)$

Special case: Exponential Functions

For the function $y = e^{u(x)}$, the derivative is
$y' = u'(x)e^{u(x)}$

Implicit Differentiation

This is used to find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ in case it is difficult or impossible to solve for y explicitly in terms of x.
In this case we will use the Chain Rule in the form

$$\frac{\mathrm{d}}{\mathrm{d}x} f(y(x)) = f'(y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$