Limits involving ∞

- Without using the graph, how to tell if $\lim_{x \to c} f(x) = \pm\infty$?

- Ex: $\lim_{x \to 3} \frac{x+2}{x-3}$

General rule: For a rational function, if the numerator → non-zero and the denominator → 0, the limit will either be ∞ or -∞ (one-sided at least)

So we look at the one-sided limits separately

$\lim_{x \to 3^-} \left(\frac{x+2}{x-3}\right) = ?$

Choose x-values close to 3, but below 3

$x = 2.5 \quad f(x) = \frac{2.5+2}{2.5-3} = \frac{4.5}{-0.5} = -9$

$x = 2.9 \quad f(x) = \frac{2.9+2}{2.9-3} = \frac{4.9}{-0.1} = -49$

It looks like

$\lim_{x \to 3^-} \left(\frac{x+2}{x-3}\right) = -\infty$

$\lim_{x \to 3^+} \left(\frac{x+2}{x-3}\right) = ?$

$x = 3.5 \quad f(x) = \frac{3.5+2}{3.5-3} = \frac{5.5}{0.5} = 11$

$x = 3.1 \quad f(x) = \frac{3.1+2}{3.1-3} = \frac{5.1}{0.1} = 51$

It looks like

$\lim_{x \to 3^+} \left(\frac{x+2}{x-3}\right) = \infty$