1) For the function $f(x)$ whose graph is given above, find each of the following values or limits. If the limit does not exist, please write "Does not exist." If the limit is ∞ or $-\infty$, say so.

a) $f(-3) = -2$

b) $\lim_{x\to -3^-} f(x) = -2$

c) $\lim_{x\to -3^+} f(x) = 0$

d) $\lim_{x\to 3} f(x)$ does not exist

e) $f(4) = 1$

f) $\lim_{x\to 4} f(x) = 2$

g) $\lim_{x\to 2^-} f(x) = -\infty$

h) $\lim_{x\to 2^+} f(x) = 3$

i) $f(0)$ is undefined

j) $\lim_{x\to 0} f(x) = \infty$

2) For the function $f(x) = \frac{\sin x}{x}$, find each of the values to complete the table:

x	f(x) to 6 decimal places	x	f(x) to 6 decimal places
1	0.841471	-1	- 0.841471
0.1	0.998334	-0.1	- 0.998334
0.01	0.999983	-0.01	- 0.999983
0.001	1.000000	-0.001	-1.000000
0.0001	1.000000	-0.0001	-1.000000

The limit seems to be 1. The values of the function are getting very close to 1 as x gets close to 0.

3) a) We cannot find this limit directly by substitution, because both the numerator and denominator are 0 when we substitute in x=2. However, we can factor the numerator and denominator and use the theorem about limits of functions that agree except at one point:

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x^2+x-6} &= \lim_{x\to 2} \frac{(x+2)(x-2)}{(x+3)(x-2)} \\ &= \lim_{x\to 2} \frac{x+2}{x+3} \\ &= \frac{2+2}{2+3} \\ &= \frac{4}{5}\end{aligned}$$

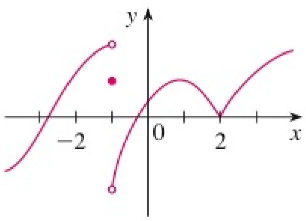
b) Here we can find the limit by substitution because this is a limit of a rational function and -2 is in the domain of that function:

$$\begin{aligned}\lim_{x\to -2} \frac{x^2-4}{x^2+x-6} &= \frac{(-2)^2-4}{(-2)^2+(-2)-6} \\ &= \frac{0}{-4} \\ &= 0\end{aligned}$$

c) $\lim_{x\to -3^+} \frac{x^2-4}{x^2+x-6}$ cannot be found by substitution, because the denominator is 0 when we substitute. Also, we cannot use the theorem about functions which agree except at a point, because there is no factor in the numerator to cancel the factor (x+3) in the denominator. This limit does not exist: by examining values of the function for x getting close to -3 from the right, we can say more.

x	f(x)
-3.5	3
-3.1	11
-3.01	101
-3.001	1001
-3.0001	10001

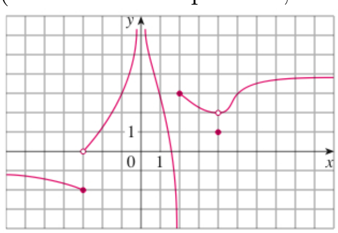
The value of the function is growing without bound. So $\lim_{x \to -3^+} \dfrac{x^2 - 4}{x^2 + x - 6} = \infty$



4)

$f(x)$ is not continuous at these values of x:
$x = -1$: a jump discontinuity

(Note: for more practice, answer this question for the graph in problem #1:)



$f(x)$ is not continuous at these values of x:
$x = -3$: a jump discontinuity
$x = 0$: an infinite discontinuity
$x = 2$: an infinite discontinuity (on the left, at least)
$x = 4$: a removable discontinuity

(Note: for yet more practice, see the WeBWorK assignment “Limits-Continuity2”.)