Definition of the Derivative of a Function

Sybil Shaver

February 26, 2018

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$
- slope of secant line $= \frac{f(c+h)-f(c)}{h}$

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$
- slope of secant line $= \frac{f(c+h)-f(c)}{h}$
- We define the derivative of f when $x = c$ to be the limit of these slopes as h goes to 0, if that limit exists:

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$
- slope of secant line $= \frac{f(c+h)-f(c)}{h}$
- We define the derivative of f when $x = c$ to be the limit of these slopes as h goes to 0, if that limit exists:
- $f'(c) = \lim_{h \to 0} \dfrac{f(c+h) - f(c)}{h}$

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$
- slope of secant line $= \frac{f(c+h)-f(c)}{h}$
- We define the derivative of f when $x = c$ to be the limit of these slopes as h goes to 0, if that limit exists:
- $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$
- If $f'(c)$ exists, we say that f is **differentiable** at c.

Defining the derivative of a function at a point

If $f(x)$ is continuous on some open interval, and c is a point in that interval,

- The average rate of change of f between $x = c$ and where $x = c + h$ is the slope of the secant line to the graph of f between the points where $x = c$ and where $x = c + h$
- slope of secant line $= \frac{f(c+h)-f(c)}{h}$
- We define the derivative of f when $x = c$ to be the limit of these slopes as h goes to 0, if that limit exists:
- $f'(c) = \lim_{h \to 0} \dfrac{f(c+h) - f(c)}{h}$
- If $f'(c)$ exists, we say that f is **differentiable** at c.
- If $f'(c)$ exists at every point in the open interval, we say that f is **differentiable** on that interval.

Defining the tangent line to the graph of a function at a point

If $f(x)$ is differentiable at c,

- The tangent line to the graph of $f(x)$ at c is the line which passes through the point $(c, f(c))$ and has slope $f'(c)$.

Defining the tangent line to the graph of a function at a point

If $f(x)$ is differentiable at c,

- The tangent line to the graph of $f(x)$ at c is the line which passes through the point $(c, f(c))$ and has slope $f'(c)$.
- The tangent line at c has the equation $y = f'(c)(x - c) + f(c)$

Defining the derivative as a function

If $f(x)$ is differentiable on some open interval,

- The derivative of f is the function $f'(x)$ defined by

Defining the derivative as a function

If $f(x)$ is differentiable on some open interval,

- The derivative of f is the function $f'(x)$ defined by
- $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Defining the derivative as a function

If $f(x)$ is differentiable on some open interval,

- The derivative of f is the function $f'(x)$ defined by
- $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- There are some other notations used for the derivative function: if $y = f(x)$,

Defining the derivative as a function

If $f(x)$ is differentiable on some open interval,

- The derivative of f is the function $f'(x)$ defined by
- $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- There are some other notations used for the derivative function: if $y = f(x)$,
- $f'(x)$ is also denoted by y' or $\frac{\mathrm{d}f}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(f)$ or $\frac{\mathrm{d}}{\mathrm{d}x}(y)$.