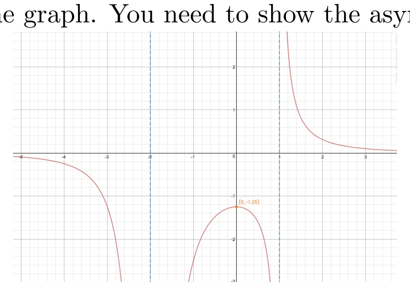
For more information and practice, and also for the graphs, see the course blog and the Piazza discussion board.

Self-Test A

1)

$$f(x) = \frac{5}{(x+2)^2(x-1)}$$

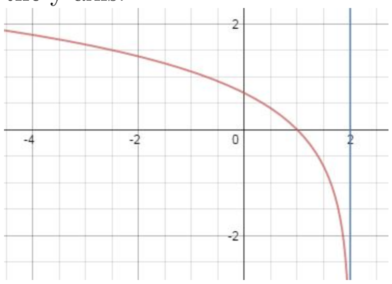
The domain is $\mathbb{R} \setminus \{-2, 1\}$ [from looking at the zeroes of the denominator]
The y-intercept is $(-\frac{5}{4}, 0)$ [Compute $f(0)$]
There is no x-intercept because the numerator of this function is never 0 so there is no point where $f(x) = 0$.
There are vertical asymptotes at $x = -2$ and $x = 1$
There is a horizontal asymptote at $y = 0$ (the x-axis) because the degree of the numerator is less than the degree of the denominator.
Sketch all of these things on the graph paper before you look at your calculator to get an idea of the shape of the graph. You need to show the asymptotes (as dotted or dashed lines).
The graph: 

2) First solve the corresponding equation: $\frac{x^2-4x}{2x^2-18} = 0$
The solutions are from the numerator being zero: $x^2 - 4x = 0$ which can be solved by factoring. This gives $x = 0$, $x = 4$ as solutions of the corresponding equation.
Then find the zeroes of the denominator (which give vertical asymptotes or holes): $2x^2 - 18 = 0$ can also be solved by factoring to give $x = 3$, $x = -3$ as the equations of the vertical asymptotes. (There are no holes.)
Use these four x-values to cut the real line into 5 intervals and test a point in the interior of each of those intervals, or else look at the graph of the function (if you do this, sketch the graph as a way of showing your work). Then check whether the endpoints are included or not. Here the endpoints at -3 and 3 are not included, because the rational function is undefined there. The endpoints at 0 and 4 are included, because the rational function equals 0 there. So the solution set for the inequality is
$(-3, 0] \cup (3, 4]$

3) First solve the corresponding equation; the three solutions are $x = 0$, $x = 2$, $x = 3$. Then either choose a test point inside each of the intervals $(-\infty, 0)$, $(0, 2)$, $(2, 3)$, and $(3, \infty)$ and check to see if the inequality holds there: or else look at the graph of $y = x^3 - 5x^2 + 6x$ and see where the graph lies above the x-axis. Finally, notice that the endpoints are not included in the solution set. The solution set is
$(0, 2) \cup (3, \infty)$

4) The domain is $(-\infty, 2)$
The x-intercept is $(1, 0)$
The vertical asymptote is $x = 2$
The graph is the same as the graph of $y = \log(x)$, shifted to the right by 2 units and reflected over

the y-axis.



5) $\log\left(\frac{\sqrt{x^3}}{y^5}\right) = \log\left(\frac{x^{3/2}}{y^5}\right) = \log(x^{3/2}) - \log(y^5) = \frac{3}{2}\log(x) - 5\log(y) = \frac{3}{2}u - 5v$

6) You have to solve $100\,(0.985^t) = 60$, which gives $t = \frac{\ln(0.6)}{\ln 91.015)} \approx 33.8$ years: so that will be late in the year 2051.

Note: please make sure that you are using the correct formula as we used in class. The formula does NOT involve the base e.

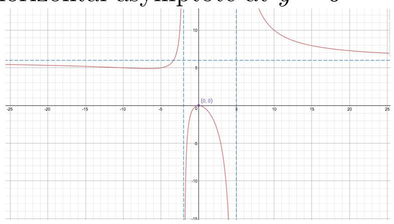
Self-Test B

1) The domain is $\mathbb{R} \setminus \{-2, 5\}$

The x-intercept is the same as the y-intercept : $(0, 0)$

There are two vertical asymptotes: $x = -2$ and $x = 5$

There is a horizontal asymptote at $y = 6$

The graph: 

2) $(-\infty, -2) \cup \{0\} \cup (5, \infty)$

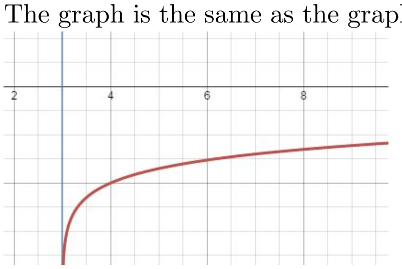
3) $(-\infty, 0)$

4) The domain is $(3, \infty)$

The x-intercept is $(103, 0)$

The vertical asymptote is $x = 3$

The graph is the same as the graph of $y = \log(x)$, shifted to the right by 3 units and down by 2 units.



5) This is similar to problem (2) in the previous Self-Test. The final answer is $\frac{3}{2}u - \frac{1}{2}v$.

6 a) $f(t) = 100\left(\sqrt{15}\right)^t$ or you can write it as $f(t) = 100\,(15)^{t/2}$

b) $f(4) = 22500$

c) $t \approx 5.1$ months