Complex numbers in polar form

A complex number written in the form $z = a + bi$ is said to be in **rectangular form**.
In **polar form** we write the complex number in the form $r(\cos\theta + i\sin\theta)$
Shorthand: r cis θ

where r is the **modulus** of the complex number: $r = |a + bi| = \sqrt{a^2 + b^2}$
and θ is the **argument** of the complex number, which is the angle formed between the positive real axis and the ray from (0,0) through (a,b).
To return to rectangular form:
$a = r\cos\theta$, $b = r\sin\theta$

To multiply two complex numbers in polar form: multiply the moduli and **add the angles**:
Example 1:

$$\begin{aligned}[2(\cos(150^\circ) + i\sin(150^\circ))] \cdot [3(\cos(60^\circ) + i\sin(60^\circ))] &= 2 \cdot 3(\cos(150^\circ + 60^\circ) + i\sin(150^\circ + 60^\circ)) \\ &= 6(\cos(210^\circ) + i\sin(210^\circ))\end{aligned}$$

Often we will want to change this result into standard form $a + bi$. This is easy to do, we just find the values of the sine and cosine, substitute them in, and use the distributive law.
Notice that we know from the unit circle that $\cos(210^\circ) = -\frac{\sqrt{3}}{2}$ and $\sin(210^\circ) = -\frac{1}{2}$.

$$\begin{aligned}6(\cos(210^\circ) + i\sin(210^\circ)) &= 6\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) \\ &= -\frac{6\sqrt{3}}{2} - i\frac{6}{2} \\ &= -3\sqrt{3} - 3i\end{aligned}$$

To divide: divide the moduli and **subtract the angles**:
Example:

$$\begin{aligned}\frac{10(\cos(120^\circ) + i\sin(120^\circ))}{2(\cos(30^\circ) + i\sin(30^\circ))} &= \frac{10}{2}(\cos(120^\circ - 30^\circ) + i\sin(120^\circ - 30^\circ)) \\ &= 5(\cos(90^\circ) + i\sin(90^\circ))\end{aligned}$$

If we want this result to be put in standard form $a + bi$, we recall from the unit circle that $\cos(90^\circ = 0$ and $\sin(90^\circ = 1$:
$5(\cos(90^\circ) + i\sin(90^\circ)) = 5(0 + i \cdot 1) = 0 + 5i$ or just $5i$