1) For the functions $f(x) = \frac{1}{5x+2}$ and $g(x) = x - 2$, find and simplify each of the following:

a) $f \circ g(x)$

$$\begin{aligned} f \circ g(x) &= f\left(g(x)\right) \\ &= f(x-2) \\ &= \frac{1}{5(x-2)+2} \\ &= \frac{1}{5x-10+2} \\ &= \frac{1}{5x-8} \end{aligned}$$

b) $g \circ f(x)$

$$\begin{aligned} g \circ f(x) &= g\left(f(x)\right) \\ &= g(\frac{1}{5x+2}) \\ &= \frac{1}{5x+2} - 2 \\ &= \frac{1}{5x+2} - \frac{2(5x+2)}{5x+2} \\ &= \frac{1-10x-4}{5x+2} \\ &= \frac{-10x-3}{5x+2} \end{aligned}$$

c) From the results of (a) and (b), using the "round-trip" theorem, are $f(x)$ and $g(x)$ inverse functions? Explain. (**Do not do any more algebra to answer this.** Just use the results of parts (a) and (b).)
Answer: f and g are not inverse functions, because the round-trip theorem says that for inverse functions we must have $f \circ g(x) = x$ and $g \circ f(x) = x$, and neither of those things is true, as we showed above: $f \circ g(x) = \frac{1}{5x-8} \neq x$, and $g \circ f(x) = \frac{-10x-3}{5x+2} \neq x$.
Note: when the problem told you not to do any more algebra, it was specifying that you not use anything other than the results of parts (a) and (b). You were specifically told to use the "round-trip" theorem. Any other way of answering this question was incorrect.

2) Is each of the following functions even, odd, or neither even nor odd? Explain.

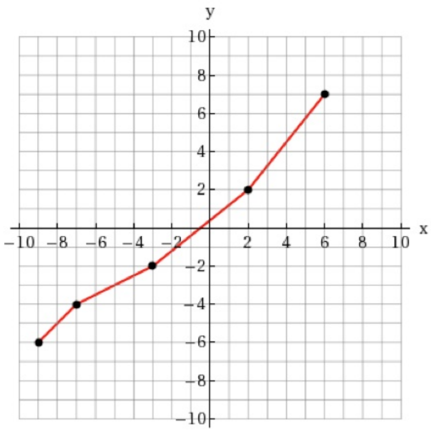
a) $f(x) = 3 - x$
This is neither:
$f(-x) = 3 - (-x) = 3 + x \neq f(x)$, so it is not even
and $-f(x) = -(3-x) = -3 + x \neq f(-x)$, so it is not odd.

b) $g(x) = 5x^3 - 2x$
$g(-x) = 5(-x)^3 - 2(-x) = -5x^3 + 2x$
$-g(x) = -(5x^3 - 2x) = -5x^3 + 2x = g(-x)$, so it is odd.
It is also correct to say that both powers of x are odd powers, so this is an odd function. However, be careful! If you tried to use this kind of reasoning in part (a), you need to remember that a constant term has degree 0, which is even. Many people forget to think about the constant term!

3) Below is the graph of a function $f(x)$:

a) How do we know from the graph itself that it is the graph of a function?
Answer: the graph passes the vertical line test: every vertical line intersects the graph in at most one point: every input has only one output.

b) Is f(x) one-to-one? Explain.
Answer: Yes, the graph passes the horizontal line test: every horizontal line intersects the graph in at most one point: every output comes from only one input.



4) Use algebra to find the inverse function of $f(x) = 5 - 7x$
$y = 5 - 7x$
Interchange x and y: $x = 5 - 7y$
Solve for y:
$x - 5 = -7y$
$\frac{x-5}{-7} = y$
$y = -\frac{x-5}{7}$ or $\frac{5-x}{7}$ (we do not leave a negative number as denominator)
So $f^{-1}(x) = \frac{5-x}{7}$

5) a) **Without using synthetic or long division**, find the remainder when $x^5 - 4x^2 - 3x + 10$ is divided by $x - 2$. Explain, or show your work.
By the remainder theorem, the remainder will be
$p(2) = 2^5 - 4(2^2) - 3(2) + 10 = 32 - 16 - 6 + 10 = 20$

b) Using your result of (a), is $x - 2$ a factor of $x^5 - 4x^2 - 3x + 10$? Why or why not? (Do not do any more computations. Just use the result of part (a).)
No, $x - 2$ is not a factor of $x^5 - 4x^2 - 3x + 10$, because the remainder is not 0.

6) Find the quotient and remainder when $x^5 + 3x^4 - 20$ is divided by $x^2 + 3$.

$$
\begin{array}{r|l}
 & x^3 + 3x^2 - 3x - 9 \\
\hline
x^2+3 & x^5 + 3x^4 \qquad\qquad\qquad - 20 \\
 & -x^5 \qquad - 3x^3 \\
\hline
 & 3x^4 - 3x^3 \\
 & -3x^4 \qquad - 9x^2 \\
\hline
 & -3x^3 - 9x^2 \\
 & 3x^3 \qquad + 9x \\
\hline
 & -9x^2 + 9x - 20 \\
 & 9x^2 \qquad + 27 \\
\hline
 & 9x + 7
\end{array}
$$

The quotient is $x^3 + 3x^2 - 3x - 9$; the remainder is $9x + 7$.
Note: The program I am using to typeset this division problem will not let me put in the 0 place-holder terms for some reason. It does leave a blank space where they should be, so we can line up the like terms: that is the important thing.
Also, this division cannot be done synthetically, because the divisor is not first degree. We have no choice but to use long division here.

7) Consider the polynomial $f(x) = x^3 + 2x^2 - 3x - 6$

a) Find one exact root of f, show that it is a root, and then use it to find all of the exact roots of f.
By looking at the graph (or just guessing possible integer roots) it looks like there is a root at -2.

Verify this by synthetic (or long) division:

$$
\begin{array}{r|rrrr}
 & 1 & 2 & -3 & -6 \\
-2 & & -2 & 0 & 6 \\
\hline
 & 1 & 0 & -3 & \boxed{0}
\end{array}
$$

The remainder is 0 as it should be: the quotient is $x^2 - 3$. To find the remaining exact roots we solve $x^2 - 3 = 0$:
$x^2 = 3$
$x = \pm\sqrt{3}$ by the square root property.
So the exact roots are: $-2, \sqrt{3}, -\sqrt{3}$

b) Use the roots you found in part (a) to factor f completely.
A root c corresponds to a factor $x - c$, so we have
$f(x) = (x + 2)(x - \sqrt{3})(x + \sqrt{3})$