

Session 7: **Inverse Functions**

Definition: If $f(x)$ is a function which is one-to-one, we define the inverse of f to be the function $f^{-1}(x)$ such that

$$y = f(x) \iff f^{-1}(y) = x$$

The domain of f is the range of its inverse, and the range of f is the domain of its inverse.

In other words: the inverse function to $f(x)$ reverses the roles of the input and output.

Example: Finding the rule (or formula) for the inverse function of $f(x) = 3x - 2$

We first rewrite the equation for the rule of $f(x)$, using $y = f(x)$:

$$y = 3x - 2$$

Now we change y to x and x to y in this equation:

$$x = 3y - 2$$

And now solve for y :

$$x = 3y - 2$$

$$x + 2 = 3y$$

$$\frac{x + 2}{3} = y$$

So the inverse function is $f^{-1}(x) = \frac{x+2}{3}$

Note: In the formula for $f(x)$, what operations do we do to x ?

What about the inverse function $f^{-1}(x)$?

Knowing the graph of $f(x)$, we can find the graph of its inverse function. Here is an example:

