

- 1) Solve the inequality and write your solution in interval form:

$$x^2 - 2x \geq 35$$

Rewrite so one side is 0: $x^2 - 2x - 35 \geq 0$ First we solve the corresponding equation:

$$\begin{aligned}x^2 - 2x - 35 &= 0 \\(x + 5)(x - 7) &= 0\end{aligned}$$

The roots are $x = -5$ and $x = 7$

They cut the real line into three intervals: $(-\infty, -5)$, $(-5, 7)$, and $(7, \infty)$

We either use a test point inside each interval, or consider the graph. Here I will do both, to show how they are connected.

In the interval $(-\infty, -5)$ I choose $x = -6$. Substituting in the inequality:

$$\begin{aligned}(-6)^2 - 2(-6) - 35 &\geq ?0 \\36 + 12 - 35 &= 13 \geq 0\end{aligned}$$

It satisfies the inequality, so not only -6, but the whole interval $(-\infty, -5)$ is in the solution set.

In the interval $(-5, 7)$ I choose $x = 0$. Substituting in the inequality:

$$\begin{aligned}(0)^2 - 2(0) - 35 &\geq ?0 \\0 - 0 - 35 &= -35 \not\geq 0\end{aligned}$$

It does not satisfy the inequality, so not only 0, but the whole interval $(-5, 7)$ is excluded from the solution set.

In the interval $(7, \infty)$ I choose $x = 10$. Substituting in the inequality:

$$\begin{aligned}(10)^2 - 2(10) - 35 &\geq ?0 \\100 - 20 - 35 &= 45 \geq 0\end{aligned}$$

It satisfies the inequality, so not only 10, but the whole interval $(7, \infty)$ is in the solution set.

Finally, consider the endpoints: since they are the solutions of the corresponding equation and our inequality is “greater than or equal to”, they are also included in the solution set.

So the solution set to the inequality is $(-\infty, -5] \cup [7, \infty)$

Or we could see this from graphing $y = x^2 - 2x - 35$ and looking at the graph to see what x-values give y-values which are greater than or equal to 0:

- 2) Solve the inequality and write your solution in interval form:

$$\frac{x-5}{2-x} \leq 0$$