MAT 1375: Functions given by formulas

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- It may be defined by giving all the values of inputs and their outputs (in a table, for example, or a mapping diagram)
- More commonly, it may be defined by a verbal description, a formula, or a graph.

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- **Otherwise, we take the domain to be the largest possible set of real numbers for which the function is defined.**
- That means in practice we must look for real numbers where the function is not defined!
- So if the function is given by a formula, we ask what could possibly go wrong.

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- So the domain is the set of all real numbers, $\mathbb{R}$

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- So we need the radicand to be non-negative:
- For this function we need $x^2 - 3 \geq 0$. Solving this polynomial inequality will give the domain.

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- What is the domain of this function? Put the parts together. It may help to draw a graph.
- The domain is the union of $[-1, 1]$ and $(1, 5]$, which is the whole interval $[-1, 5]$.

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- So we need the denominator not to be 0:
- We need $x + 3 \neq 0$. Solving this inequality will give the domain: $x \neq -3$
- We can write the domain as $\mathbb{R} \setminus \{-3\}$, which means "the set of all real numbers except -3"