## TRIGONOMETRIC IDENTITIES

Recall that:

$$\tan x = \frac{\sin x}{\cos x}$$
  $\cot x = \frac{\cos x}{\sin x}$   $\sec x = \frac{1}{\cos x}$   $\csc x = \frac{1}{\sin x}$ 

(1) Rewrite the following trigonometric functions in terms of  $\cos x$  and  $\sin x$  only. Do not perform any algebraic simplification yet.

**Example:** The function  $\cot x(\sec x + \sin x)$  can be rewritten as  $\frac{\cos x}{\sin x} \left( \frac{1}{\cos x} + \sin x \right)$ .

(a) 
$$(\tan x)(\sec x) = \left(\begin{array}{c} \\ \end{array}\right)$$

(b) 
$$1 + \tan^2 x = 1 + \left( \frac{1}{1 + \tan^2 x} \right) = (\tan x)(\tan x) = (\tan x)^2$$
  
and  $\tan^2 x \neq \tan(x^2)$ 

(c) 
$$\csc^2 x - \cot^2 x = \left(\begin{array}{c} \\ \\ \end{array}\right) - \left(\begin{array}{c} \\ \end{array}\right)$$
 [Note: don't forget to add the "squares"]

(d) 
$$\frac{\cot x}{\csc x} = \frac{\left(----\right)}{\left(----\right)}$$

(e) 
$$\cos x(\sec x - \cos x) =$$
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(f) 
$$\tan^2 x \csc^2 x - \tan^2 x = \left( - - - \right) \left( - - - \right) - \left( - - - - \right)$$

(g) 
$$\frac{(\csc x - \cot x)(\csc x + \cot x)}{\tan x} =$$

(2) For each function in question (1), substitute each  $\cos x$  by a and each  $\sin x$  by b. Do not perform any algebraic simplification yet.

**Example (cont'd):** The function 
$$\frac{\cos x}{\sin x} \left( \frac{1}{\cos x} + \sin x \right)$$
 can be rewritten as  $\frac{a}{b} \left( \frac{1}{a} + b \right)$ .

Copy here your answer from 1(a)

$$\downarrow$$

(a) 
$$\left( - - - - \right) \left( - - - - \right) = \left( - - - - \right) \left( - - - - \right)$$

Copy here your answer from 1(b)

Copy here your answer from 1(c)



$$(c) \left( \begin{array}{c} \\ \\ \end{array} \right) - \left( \begin{array}{c} \\ \\ \end{array} \right) = \left( \begin{array}{c} \\ \\ \end{array} \right) - \left( \begin{array}{c} \\ \\ \end{array} \right)$$

Copy here your answer from 1(d)



$$(\mathrm{d}) \ \frac{\left( - - - \right)}{\left( - - - \right)} = \frac{\left( - - - \right)}{\left( - - - \right)}$$

Copy here your answer from 1(e)



Copy here your answer from 1(f)

(g)

(3) Simplify each expression in question (2) algebraically.

**Example (cont'd):** The expression  $\frac{a}{b} \left( \frac{1}{a} + b \right)$  can be rewritten as

$$\frac{a}{b}\left(\frac{1}{a} + \frac{ab}{a}\right) = \frac{a}{b}\left(\frac{1+ab}{a}\right) = \frac{a(1+ab)}{ba} = \frac{1+ab}{b}.$$

Notice that the simplification led to a single fraction.

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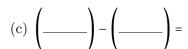


(a) 
$$\left( - - - - \right) \left( - - - - - \right) =$$

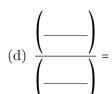
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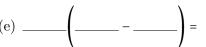
Copy here your answer from 2(c)



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Copy here your answer from 2(e)



Copy here your answer from 2(f)



Now we can finally start proving trigonometric identities. Basically, we will put together the three procedures we have just practiced: rewrite, substitute, and simplify.

**Step 1: Rewrite** the identity in terms of  $\cos x$  and  $\sin x$  only.

Step 2: Substitute each  $\cos x$  by a and each  $\sin x$  by b.

**Step 3:** Simplify each side separately. You are done when the LHS is equal to the RHS.

Remark: sometimes, in order to show that the two sides are equal, we have to use the fundamental identity:

 $\cos^2 x + \sin^2 x = 1.$ 

which can be rewritten as

which can be rewritten as  $(\cos x)^2 + (\sin x)^2 = 1$ , or  $a^2 + b^2 = 1$ . So, whenever you see  $a^2 + b^2$ , **remember** to replace it by 1. Things will be simpler!

**Example:** Show that  $\cos x + \sin x \cdot \tan x = \sec x$ . Solution:  $\cos x + \sin x \cdot \tan x = \sec x$  $\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x}$ Step 1: rewrite  $a + b \cdot \frac{b}{a} = \frac{1}{a}$ Step 2: substitute  $a + \frac{b^2}{a} = \frac{1}{a}$ Step 3: simplify turn the LHF into a single fraction  $\frac{a \cdot a}{a} + \frac{b^2}{a} = \frac{1}{a}$ Don't move the terms from one side to the other  $\frac{a^2}{a} + \frac{b^2}{a} = \frac{1}{a}$  $\frac{a^2 + b^2}{a} = \frac{1}{a}$ **Remember:**  $a^2 + b^2 = 1$ 

Done! ©

- (1) Show that:
  - (a)  $\cos x \tan x = \sin x$
  - (b)  $\csc^2 x \tan^2 x = \sec^2 x$
  - (c)  $\sin x(\csc x \sin x) = \cos^2 x$
  - (d)  $\frac{\sin x \cos x + \sin x}{\cos x + \cos^2 x} = \tan x$
  - (e)  $\frac{(\sec x + \tan x)(\sec x \tan x)}{\csc x} = \sin x$
  - (f)  $\frac{\cot x}{\sec x} \frac{\cos x}{\sin x} = \frac{\cos x 1}{\tan x}$