DCU School of Mathematical Sciences

BASIC SKILLS WORKSHEET 8

Logarithms

The aim of this worksheet is to introduce the basics of logarithms and their applications.

What are logarithms?

The equation

$$2^3 = 8$$

expresses a certain relationship between the numbers 2, 3 and 8. There is no reason why we shouldn't express this relationship in a different way. In the 17th century, the Scots mathematician John Napier discovered/invented a different and very useful way of writing this simple relationship. He defined logarithms (logs) by rewriting the equation above in the form

$$\log_2 8 = 3.$$

We read this as "log to the base 2 of 8 equals 3". That's all there is to logs, it's just a different way of expressing the relationship between certain numbers.

Let me repeat that.

That's all there is to logs, it's just a different way of expressing the relationship between certain numbers.

Every equation which can be written using an index or power (this is called an exponential form) can also be written in a corresponding logarithmic form.

Examples

- $2^3 = 8$ is the same as $\log_2 8 = 3$.
- $4^2 = 16$ is the same as $\log_4 16 = 2$
- $5^6 = 15625$ is the same as
- $3^{-2} = \frac{1}{9}$ is the same as

Note

$\log_{10}$ is used very frequently. For this reason, the 10 is usually omitted, so

$$\log = \log_{10}.$$

This is the "log" key that appears on your calculator. Practice using it; you should find that $\log 10 = 1$, $\log 100 = 2$, $\log 34.2 = 1.534026...$

Laws of logarithms

The three laws of indices can be translated into three corresponding laws of logarithms. As with the laws of indices, these can be used to simplify expressions and to shorten calculations.

1. $\log(ab) = \log a + \log b$.
2. $\log \frac{a}{b} = \log a - \log b$.
3. $\log a^m = m \log a$.

We won't bother doing a bunch of exercises simplifying certain expressions. We go straight to the reason people bother studying logs.

Example

Solve the equation

$$2^x = 15.$$

(This equation wouldn't be any problem if it were $2^x = 16$. Then it would simply be a matter of choosing the correct whole number value for x, which would be $x = 4$ in this case.) Back to $2^x = 15$. We have $2^3 = 8$ and $2^4 = 16$, so $x = 3$ is too small and $x = 4$ is too big. How do we find the value of x that is just right?

The answer is to use logs, and especially the third law. Let's take log of both sides of the equation

$$\begin{aligned} 2^x &= 15 \\ \Rightarrow \log 2^x &= \log 15 \\ \Rightarrow x \log 2 &= \log 15 \qquad \text{using the third law} \\ \Rightarrow x &= \frac{\log 15}{\log 2} \\ &= 3.91 \qquad \text{correct to 2 decimal places.} \end{aligned}$$

Exercise

1. Write down the value of
   (a) $\log_3 27$
   (b) $\log 1000$
   (c) $\log_2 \frac{1}{4}$
   (d) $\log_5 625$
2. Solve the following equations for x.
   (a) $3^x = 25$
   (b) $(1.025)^x = 2.654$
   (c) $100(1.05)^x = 1650$

Solutions to exercises

1. (a) 3
   (b) 3
   (c) -2
   (d) 4
2. (a) 2.9300
   (b) 39.5287
   (c) 57.4575