

MAT 1275 Final Exam review sheet

#1 Solve for x (simplify completely):

a) $6x^2 - 2x - 3 = 0$

b) $2x^2 - 4x + 5 = 0$

c) $2x^2 + 2x = -3$

#2 Graph the function and label the vertex and the x- and y- intercepts with numbers or coordinates:

a) $y = x^2 - 2x - 3$

b) $y = x^2 + 2x$

c) $y = -x^2 + 4x - 2$

#3 Simplify the complex fractions: a) $\frac{\frac{2}{y^2} + \frac{1}{y}}{\frac{4}{y^2} - \frac{1}{y}}$

b) $\frac{\frac{2}{x} + \frac{1}{y}}{\frac{3}{y} - \frac{4}{x}}$

c) $\frac{3 - \frac{1}{2x}}{2 + \frac{1}{x^2}}$

#4 Find the center and radius of each circle, and sketch the graph, labelling four points on the circle with coordinates:

a) $x^2 + y^2 - 6x + 4y - 4 = 0$

b) $x^2 + y^2 + 8x - 2y - 8 = 0$

c) $x^2 + y^2 + 10x + 6y + 18 = 0$

#5 Perform the indicated operations involving Complex Numbers and express in standard complex form:

a) $(-2 + 3i)(5 - 7i)$

b) $(4 - 9i)(3 - 2i)$

c) Simplify: $\frac{3 - 2i}{1 - i}$

d) Simplify: $\frac{2 - 7i}{3 + 4i}$

#6 Solve the following system of equations for x, y and z:

$3x + 2y - z = 4$
a) $2x - y + 2z = 10$
 $x + 3y - 4z = -7$

$-2x + 5y + z = 8$
b) $x - 2y - 3z = -13$
 $x + 3y - z = 5$

$2x - 3y + z = -9$
c) $3x + 5y + 2z = 16$
 $-4x + 2y - 3z = 4$

#7 Solve each of the following for x and y:

a) $x^2 + y^2 = 5$
 $x - y^2 = -3$

b) $x^2 - y^2 = 3$
 $2x + y^2 = 5$

c) $x^2 + y^2 = 8$
 $x^2 - y = 2$

#8 For the information given, find the values for the five remaining trig functions for θ :

a) $\sin\theta = \frac{2}{5}$ and $\tan\theta > 0$

b) $\tan\theta = 2$ and $\cos\theta < 0$

c) $\csc\theta = -3$ and $\cos\theta > 0$

#9 Given each $\triangle ABC$, answer each of the following (round each answer to nearest tenth):

a) If $a = 12$, $b = 8$ and $c = 5$. Find $\angle C$

b) If $\angle A = 50^\circ$, $\angle B = 75^\circ$, $a = 20$. Find side b.

c) If $b = 9$, $c = 6$ and $\angle A = 67^\circ$. Find side a.

d) If $\angle C = 37^\circ$, $\angle B = 79^\circ$, $b = 13$. Find side a.

#10 Prove each of the following Trig Identities:

a) $\cos x + \sin x \cdot \tan x = \sec x$

b) $\csc x - \sin x = \cot x \cdot \cos x$

c) $\frac{1}{\cos x} - \frac{1}{\sec x} = \tan x \cdot \sin x$

d) $\csc x \cdot \cot x \cdot \sec x = 1 + \cot^2 x$

e) $\sin x \cdot \tan x \cdot \cos x = 1 - \cos^2 x$

f) $\csc x \cdot \sec x = \cot x + \tan x$

#11 Solve the following Equations for x , where $0 \leq x < 2\pi$:

a) $4\sin x + 2 = 0$

b) $6\cos x - 3 = 0$

c) $2\tan x - 2 = 0$

d) $6\tan x = -2\sqrt{3}$

#12 Solve to the nearest tenth:

a) $3^x = 38$

b) $2^x = 19$

c) $4^x = 7$

d) $5^x = 29$

#13 Evaluate the following without the use of a calculator:

a) $\log_2 16\sqrt[3]{2}$

b) $\log_3 \frac{1}{9}$

c) $\log_4 64$

d) $\log_5 \sqrt{5}$

Answers to questions:

#1 a) $\frac{1 \pm \sqrt{19}}{6}$

b) $\frac{5 \pm \sqrt{7}}{2}$

c) $\frac{1 \pm \sqrt{6}}{5}$

#2 a) Vertex: (1, -4)

b) Vertex: (-1, -1)

c) Vertex: (2, 2)

#3 a) $\frac{2+y}{4-y}$

b) $\frac{2y+x}{3x-4y}$

c) $\frac{6x^2-x}{4x^2+2}$ or $\frac{x(6x-1)}{2(2x^2+1)}$

#4 a) Center: (3, -2); Radius: r = 3

b) Center: (-4, 1); Radius: r = 5

c) Center: (-5, -3); Radius: r = 4

#5 a) $11+29i$

b) $-6-35i$

c) $\frac{5}{2} + \frac{1}{2}i$

d) $-\frac{22}{25} - \frac{29}{25}i$

#6 a) $x=3, y=-2, z=1$

b) $x=3, y=2, z=4$

c) $x=-1, y=3, z=2$

#7 a) (-2, 1), (-2, -1), (1, 2), (1, -2)

b) (2, 1), (2, -1), $(4, \sqrt{13})$, $(4, -\sqrt{13})$

c) (-2, 2), (2, 2)

#8 a) $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$, $\sec \theta = \frac{5}{4}$, $\csc \theta = \frac{5}{3}$, $\cot \theta = \frac{4}{3}$

b) $\sin \theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$, $\cos \theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$, $\sec \theta = -\sqrt{5}$, $\csc \theta = -\frac{\sqrt{5}}{2}$, $\cot \theta = \frac{1}{2}$

c) $\sin \theta = -\frac{1}{3}$, $\cos \theta = \frac{2\sqrt{2}}{3}$, $\sec \theta = \frac{3\sqrt{2}}{4}$, $\tan \theta = -\frac{1}{2\sqrt{2}}$, $\cot \theta = -2\sqrt{2}$

#9 a) $\angle C = 17.6^\circ$

b) $b = 25.2$

c) $a = 8.6$

d) $a = 11.9$

#10 LHS = Left Hand Side. RHS = Right Hand Side.

a) LHS: $\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \left(\frac{\cos x}{\cos x}\right) \cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

b) LHS: $\csc x - \sin x = \frac{1}{\sin x} - \sin x \left(\frac{\sin x}{\sin x}\right) = \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \frac{\cos x}{\sin x} \cdot \cos x = \cot x \cdot \cos x$

c) LHS: $\frac{1}{\cos x} - \frac{1}{\sec x} = \frac{1}{\cos x} - \frac{\cos x}{1} \left(\frac{\cos x}{\cos x}\right) = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \cdot \sin x = \tan x \cdot \sin x$

d) LHS: $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{2 \sin x \cdot \cos x}{\sin x} - \frac{2 \cos^2 x - 1}{\cos x} = 2 \cos x - \left(\frac{2 \cos^2 x}{\cos x} - \frac{1}{\cos x}\right) = 2 \cos x - \left(2 \cos x - \frac{1}{\cos x}\right) = \frac{1}{\sec x}$

e) RHS: $\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \tan x}{\sec^2 x} = \frac{2 \sin x}{\cos x} \div \frac{1}{\cos^2 x} = \frac{2 \sin x}{\cos x} \cdot \frac{\cos^2 x}{1} = 2 \sin x \cdot \cos x = \sin 2x$

f) LHS: $\frac{\cos 2x}{\sin x \cdot \cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x} = \frac{\cos^2 x}{\sin x \cdot \cos x} - \frac{\sin^2 x}{\sin x \cdot \cos x} = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x$

Answers to questions (continued):

#11 a) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $x = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $x = \frac{\pi}{4}, \frac{5\pi}{4}$

d) $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

#12 a) $x = 3.3$

b) $x = 4.2$

c) $x = 1.4$

d) $x = 2.1$

#13 a) $4\frac{1}{3} = \frac{13}{3}$

b) -2

c) 3

d) $\frac{1}{2}$