Rationalize the denominators and simplify. Write complex numbers in the standard form $a + bi$

1) One way, rationalizing the denominator first and then simplifying:

$$\begin{aligned}
\frac{15}{\sqrt{27}} &= \frac{15}{\sqrt{27}} \cdot \frac{\sqrt{27}}{\sqrt{27}} \\
&= \frac{15\sqrt{27}}{\left(\sqrt{27}\right)^2} \\
&= \frac{15\sqrt{27}}{27} \text{ because } \left(\sqrt{27}\right)^2 = 27 : \text{ that is the definition of the square root!} \\
&= \frac{5\sqrt{27}}{9} \\
&= \frac{5\sqrt{9\cdot 3}}{9} \\
&= \frac{5\cdot 3\sqrt{3}}{9} \\
&= \frac{5\sqrt{3}}{3}
\end{aligned}$$

Another way, simplifying the denominator first:

$$\begin{aligned}
\frac{15}{\sqrt{27}} &= \frac{15}{\sqrt{9\cdot 3}} = \frac{15}{3\sqrt{3}} \\
&= \frac{5}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
&= \frac{5\sqrt{3}}{\left(\sqrt{3}\right)^2} \\
&= \frac{5\sqrt{3}}{3} \text{ because } \left(\sqrt{3}\right)^2 = 3 : \text{ that is the definition of the square root!}
\end{aligned}$$

2) Multiply top and bottom by the conjugate of the denominator:

$$\begin{aligned}
\frac{72}{\sqrt{13}-2} &= \frac{72}{\left(\sqrt{13}-2\right)} \cdot \frac{\left(\sqrt{13}+2\right)}{\left(\sqrt{13}+2\right)} \\
&= \frac{72\left(\sqrt{13}-2\right)}{\left(\sqrt{13}\right)^2 - 2^2} \quad \text{using the pattern for the product of conjugates} \\
&= \frac{72\left(\sqrt{13}-2\right)}{13-4} \\
&= \frac{72\left(\sqrt{13}-2\right)}{9} \\
&= 8\left(\sqrt{13}-2\right) \\
&= 8\sqrt{13}-16
\end{aligned}$$

3) Multiply top and bottom by the conjugate of the denominator, and remember that $i^2 = -1$:

$$\begin{aligned}
\frac{-1+6i}{8-6i} &= \frac{(-1+6i)}{(8-6i)} \cdot \frac{(8+6i)}{(8+6i)} \\
&= \frac{-8+6i+48i-36i^2}{8^2-(6i)^2} \\
&= \frac{-8+54i-36(-1)}{8^2-36i^2} \\
&= \frac{-8+54i+36}{64-36(-1)} \\
&= \frac{28+54i}{64+36} \\
&= \frac{28+54i}{100} \\
&= \frac{28}{100} + \frac{54}{100}i \\
&= \frac{7}{25} + \frac{27}{50}i
\end{aligned}$$

4) Find two angles, one positive and one negative, which are coterminal with $\frac{2\pi}{3}$. You must show or explain how you got your answers! Do all your work in radians only.

There are infinitely many possible correct answers to this - do you understand why?

You can get a coterminal angle by making a full rotation around the circle one or more times, in either the positive (counterclockwise) direction, or the negative (clockwise) direction, and then returning to the terminal side of $\frac{2\pi}{3}$. Each full rotation either adds 2π to the original angle $\frac{2\pi}{3}$, if the rotation is positive, or subtracts 2π, if the rotation is negative. So here are some possibilities: draw them and make sure that you understand what we are doing here!

Positive angles which are coterminal with $\frac{2\pi}{3}$:

$\frac{2\pi}{3} + 2\pi = \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$

$\frac{2\pi}{3} + 2(2\pi) = \frac{2\pi}{3} + 4\pi = \frac{2\pi}{3} + \frac{12\pi}{3} = \frac{14\pi}{3}$

$\frac{2\pi}{3} + 3(2\pi) = \frac{2\pi}{3} + 6\pi = \frac{2\pi}{3} + \frac{18\pi}{3} = \frac{20\pi}{3}$

$\frac{2\pi}{3} + 4(2\pi) = \frac{2\pi}{3} + 8\pi = \frac{2\pi}{3} + \frac{24\pi}{3} = \frac{26\pi}{3}$

... and so on ...

Negative angles which are coterminal with $\frac{2\pi}{3}$:

$\frac{2\pi}{3} - 2\pi = \frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$

$\frac{2\pi}{3} - 2(2\pi) = \frac{2\pi}{3} - 4\pi = \frac{2\pi}{3} - \frac{12\pi}{3} = -\frac{10\pi}{3}$

$\frac{2\pi}{3} - 3(2\pi) = \frac{2\pi}{3} - 6\pi = \frac{2\pi}{3} - \frac{18\pi}{3} = -\frac{16\pi}{3}$

$\frac{2\pi}{3} - 4(2\pi) = \frac{2\pi}{3} - 8\pi = \frac{2\pi}{3} - \frac{24\pi}{3} = -\frac{22\pi}{3}$

... and so on ...