

In this lesson, we will only be dealing with square roots. Most of these rules work for higher roots as well, as long as the indices (or degrees) of the roots are all the same.

Adding and subtracting radicals; These work even when the radicand is negative. Always simplify all the radicals **before** you try to add or subtract.

Two terms containing radicals can be combined only if they are “like terms”: this means that the radical factors in the terms must be identical. Combine them by adding or subtracting the coefficients of the radical, just as we combine like terms in algebra.

Examples:

- $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$

- $-5\sqrt{3} + \sqrt{12}$: these are not like terms. But remember that we should simplify the radicals before trying to add:

$$-5\sqrt{3} + \sqrt{12} = -5\sqrt{3} + \sqrt{4 \cdot 3} = -5\sqrt{3} + \sqrt{4} \cdot \sqrt{3} = -5\sqrt{3} + 2\sqrt{3} = -3\sqrt{3}$$

- $\sqrt{2} - \sqrt{7}$ are not like terms, and cannot be simplified: they cannot be combined.

Multiplying radicals; This works **only** when the radicands are ≥ 0 . It is not necessary to simplify the radicals before you multiply, except in the case of negative radicand, as shown in the examples. Always simplify as a last step.

$$\sqrt{A}\sqrt{B} = \sqrt{AB}$$

only when both $A \geq 0$ and $B \geq 0$

Examples:

- $\sqrt{10}\sqrt{6} = \sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$

- $\sqrt{8}\sqrt{50} = \sqrt{400} = 20$ (Try doing this by simplifying the radicals first: it is a bit more complicated!)

- $\sqrt{-4}\sqrt{-9}$: We cannot use the rule because the radicands are negative. However:

$$\sqrt{-4}\sqrt{-9} = i\sqrt{4} \cdot i\sqrt{9} = 2i \cdot 3i = 6i^2 = 6(-1) = -6$$

Dividing radicals: This works **only** when the radicands are ≥ 0 . It is not necessary to simplify the radicals before you divide, except in the case of negative radicand, as shown in the examples. Always simplify as a last step.

$$\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$$

only when both $A \geq 0$ and $B > 0$

Examples:

- $\frac{\sqrt{300}}{\sqrt{3}} = \sqrt{\frac{300}{3}} = \sqrt{100} = 10$

- $\frac{\sqrt{45}}{\sqrt{48}} = \sqrt{\frac{45}{48}} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{\sqrt{16}} = \frac{\sqrt{15}}{4}$

- $\frac{\sqrt{300}}{\sqrt{-3}}$: We cannot use the rule because one of the radicands is negative. We will discuss this example and see how to handle it when we discuss rationalizing denominators.