In these problems, do not use decimals in your answers: only use fractions or radicals in fully simplified form. Write complex numbers in the form $a + bi$

1a) Find the solutions of the equation $2x^2 - 5x - 1 = 0$.
Using the quadratic formula:

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{5 \pm \sqrt{25+8}}{4} \\ &= \frac{5 \pm \sqrt{33}}{4} \end{aligned}$$

You could also find the solutions by completing the square and using the Square Root Property, although that was not the intended method: this is much harder!

$$\begin{aligned} 2x^2 - 5x - 1 &= 0 \\ 2x^2 - 5x &= 1 \\ \frac{2x^2}{2} - \frac{5x}{2} &= \frac{1}{2} \\ x^2 - \frac{5}{2}x &= \frac{1}{2} \\ x^2 - \frac{5}{2}x + \frac{25}{16} &= \frac{1}{2} + \frac{25}{16} = \frac{8}{16} + \frac{25}{16} = \frac{33}{16} \\ \left(x - \frac{5}{4}\right)^2 &= \frac{33}{16} \\ x - \frac{5}{4} &= \pm\sqrt{\frac{33}{16}} = \pm\frac{\sqrt{33}}{4} \\ x &= \frac{5}{4} \pm \frac{\sqrt{33}}{4} \\ &= \frac{5 \pm \sqrt{33}}{4} \end{aligned}$$

1b) Find the solutions of the equation $0 = x^2 - 4x + 1$. Using the quadratic formula:

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16-4}}{2} \\ &= \frac{4 \pm \sqrt{12}}{2} \\ &= \frac{4 \pm 2\sqrt{3}}{2} \\ &= \frac{4}{2} \pm \frac{2\sqrt{3}}{2} \\ &= 2 \pm \sqrt{3} \end{aligned}$$

You could also find the solutions by completing the square and using the Square Root Property, although that was not the intended method:

$$\begin{aligned} x^2 - 4x + 1 &= 0 \\ x^2 - 4x &= -1 \\ x^2 - 4x + 4 &= -1 + 4 \\ (x-2)^2 &= 3 \\ x - 2 &= \pm\sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$$

2) For the parabola with equation $y = x^2 + 2x - 3$,

a) Find the coordinates of the vertex.
One way, using the vertex formula:
$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$ for the vertex: substitute this back into the equation of the parabola to find the y-coordinate.
$y = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$
So the vertex is the point $(-1, -4)$.

Another way, by putting the equation into "vertex form":

$$\begin{aligned} y &= x^2 + 2x - 3 \\ y + 3 &= x^2 + 2x \\ y + 3 + 1 &= x^2 + 2x + 1 \\ y + 4 &= (x+1)^2 \\ y &= (x+1)^2 - 4 \end{aligned}$$

So $h = -1$, $k = -4$, and the vertex is the point $(-1, -4)$

b) What is the equation of the axis of symmetry?
The axis of symmetry is a vertical line and it passes through the vertex $(-1, -4)$, so the equation of the axis of symmetry is $x = -1$.

c) Find the x-intercepts.
The x-intercepts are the points (if any) on the graph where $y = 0$:
$0 = x^2 + 2x - 3$
$0 = (x + 3)(x - 1)$
$x + 3 = 0$ or $x - 1 = 0$ (Zero Product Property)
$x = -3$ or $x = 1$
The x-intercepts are the points $(-3, 0)$ and $(1, 0)$

d) Find the y-intercept.
The y-intercept is the point on the graph where $x = 0$:
$y = 0^2 + 2(0) - 3 = -3$
The y-intercept is the point $(0, -3)$

3)

a) Put the equation of the circle into its standard form $(x-H)^2+(y-K)^2=R^2$:
$x^2+y^2+10x+6y+18=0$

$$\begin{aligned} x^2+y^2+10x+6y+18 &= 0 \\ x^2+10x+y^2+6y &= -18 \\ x^2+10x+25+y^2+6y+9 &= -18+25+9 \\ (x+5)^2+(y+3)^2 &= 16 \end{aligned}$$

What are the center and radius of the circle?
center: $(-5,-3)$
radius: $\sqrt{16}=4$

b) Sketch the circle on the graph paper below, and find the coordinates of the four cardinal points (top, bottom, left, and right).
Sorry, no graph here. But the four cardinal points are $(-5,1)$, $(-5,-7)$, $(-9,-3)$, and $(-1,-3)$

4) $(4,3)$ and $(-1,-9)$ are the endpoints of the diameter of a circle.
Find the center of the circle:
The center is the midpoint of a diameter, so its coordinates are the averages of the coordinates of the endpoints of the diameter:
$\left(\frac{4+(-1)}{2},\frac{3+(-9)}{2}\right)=\left(\frac{3}{2},\frac{-6)}{2}\right)=\left(\frac{3}{2},-3\right)$

Find the radius of the circle:
The radius is the distance from the center to a point on the circle, or alternatively, the radius is half of the length of the diameter. There are several ways to compute the radius from the information we have. Here is one: find the diameter and then divide it by 2: from the center $\left(\frac{3}{2},-3\right)$ to the point $(-4,1)$
$diameter=\sqrt{(4-(-1))^2+(3-(-9))^2}=\sqrt{(5)^2+(12)^2}=\sqrt{25+144}=\sqrt{169}=13$
The radius is $\frac{13}{2}$

Give the equation of the circle in its standard form $(x-H)^2+(y-K)^2=R^2$
$\left(x-\frac{3}{2}\right)^2+(y-(-3))^2=\left(\frac{13}{2}\right)^2$
Simplify to
$\left(x-\frac{3}{2}\right)^2+(y+3)^2=\frac{169}{4}$

5) Find the equation of the perpendicular bisector of the line segment that has endpoints $(5,12)$ and $(11,6)$. Put your equation into the form $y=mx+b$.
The perpendicular bisector is the line which passes through the midpoint of the given segment and which is perpendicular to that segment.

Find the midpoint of the line segment: $\left(\frac{5+11}{2},\frac{12+6}{2}\right)=\left(\frac{16}{2},\frac{18}{2}\right)=(8,9)$

Find the slope of the line segment: $m=\frac{12-6}{5-11}=\frac{6}{-6}=-1$
The slope of the perpendicular bisector is 1.

The perpendicular bisector has equation of the form $y=x+b$ and passes through $(8,9)$;
$9=8+b$
$1=b$

So the equation of the perpendicular bisector is $y = x + 1$

6) Solve the nonlinear system: give your answers as points.
$x^2 + y^2 = 5$
$x - y^2 = -3$

There are several ways to solve this system. Here is one:
We can eliminate y by adding the two equations together:

$$\begin{aligned} x^2 + y^2 &= 5 \\ x - y^2 &= -2 \\ \hline x^2 + x &= 2 \end{aligned}$$

Solve $x^2 + x = 2$ for x:
$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$(x + 2) = 0$ or $(x - 1) = 0$
$x = -2$ or $x = 1$

Now find the y-velues that go with each of those x-values: Choose either equation (I choose the first)
If $x = -2$
$(-2)^2 + y^2 = 5$
$4 + y^2 = 5$
$y^2 = 1$
$\implies y = \pm\sqrt{1} = \pm 1$ [The Square Root Property]
So there are two solutions where $x = -2$: $(-2, 1)$ and $(-2, -1)$

If $x = 1$
$1^2 + y^2 = 5$
$1 + y^2 = 5$
$y^2 = 4$
$\implies y = \pm\sqrt{4} = \pm 2$ [The Square Root Property]
So there are two more solutions where $x = 1$: $(1, 2)$ and $(1, -2)$

Solutions of the system: $(2, 1)$, $(2, -1)$, $(1, 2)$ and $(1, -2)$