

In these problems, do not use decimals in your answers: only use fractions or radicals in fully simplified form. Write complex numbers in the form  $a + bi$

- 1a) Find the equation of the line which passes through the points  $(2, -3)$  and  $(0, 1)$ . Put it in slope-intercept form.

$$\text{The slope of this line is } \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{1 - (-3)}{0 - 2} = \frac{4}{-2} = -2$$

So we know that the equation of the line has the form  $y = -2x + b$ , and from the information given in the problem the y-intercept is the point  $(0, 1)$ .

$$\text{The equation of the line is } y = -2x + 1$$

- 1b) Find the equation of the line parallel to the line in part (a), which passes through the point  $(2, -2)$ . Put it in slope-intercept form.

This line has the same slope as the line in part (a), namely,  $-2$ . So its equation has the form  $y = -2x + b$ .

This time we have not been given the y-intercept, so we use the fact that  $(2, -2)$  is a point on the line to find  $b$ :

$$-2 = -2(2) + b$$

$$-2 = -4 + b$$

$$2 = b$$

So the equation of the line is  $y = -2x + 2$

- 1c) Find the equation of the line perpendicular to the line in part (a), which passes through the point  $(-3, -1)$ . Put it in slope-intercept form.

The slope of the perpendicular line is the negative reciprocal of  $-2$ , which is  $-\frac{1}{-2} = \frac{1}{2}$

So the equation of this line has the form  $y = \frac{1}{2}x + b$ , and we use the given point to find  $b$ .

$$-1 = \frac{1}{2}(-3) + b$$

$$-1 = -\frac{3}{2} + b$$

$$-1 + \frac{3}{2} = b$$

$$-\frac{1}{2} = b$$

So the equation of this line is  $y = \frac{1}{2}x - \frac{1}{2}$

- 2) **Solve the system of equations. Indicate your final answer clearly, and check your answer.**

$$\begin{aligned} x - y - 2z &= -2 \\ 2x - 3y - 3z &= -1 \\ -2x - y + z &= -5 \end{aligned}$$

There are many ways to solve this system, but in all of the efficient ways we follow the same strategy:  
 Use any two equations to eliminate one of the variables  
 Use a different pair of equations to eliminate the same variable  
 Solve the resulting 2 by 2 system  
 Substitute back into one of the original equations to find the third variable (the one eliminated in the first step)

Here is one way:

Use the first and second equations to eliminate  $x$ :

$$\left. \begin{aligned} -2(x - y - 2z) &= -2(-2) \\ 2x - 3y - 3z &= -1 \end{aligned} \right\} \begin{aligned} -2x + 2y + 4z &= 4 \\ 2x - 3y - 3z &= -1 \\ \hline -y + z &= 3 \end{aligned}$$

Now use the first and third equations to eliminate  $x$  again:

$$\begin{array}{r}
 2(x - y - 2z) = 2(-2) \\
 -2x - y + z = -5 \\
 \hline
 -3y - 3z = -9
 \end{array}
 \left.
 \begin{array}{l}
 \\
 \\
 \end{array}
 \right\}
 \begin{array}{r}
 2x - 2y - 4z = -4 \\
 -2x - y + z = -5 \\
 \hline
 -3y - 3z = -9
 \end{array}$$

That result can be divided on both sides by  $-3$ :

$$\begin{array}{r}
 \frac{-3y}{-3} + \frac{-3z}{-3} = \frac{-9}{-3} \\
 y + z = 3
 \end{array}$$

Now we have a 2 by 2 system:

$$-y + z = 3$$

$$y + z = 3$$

Add those equations together to eliminate  $y$  and we get  $2z = 6 \implies z = 3$

Substitute back into one of the equations in the 2 by 2 system to find  $y$ : I will use the second equation

$$y + 3 = 3 \implies y = 0$$

Now choose one of the equations in the original 3 by 3 system to find  $x$ : I will use the first one

$$x - 0 - 2(3) = -2$$

$$x - 6 = -2$$

$$x = 4$$

The solution to the 3 by 3 system is  $(4, 0, 3)$

3) Factor  $49x^{12} + 14x^8 = 7x^8(7x^4 + 2)$

4) Factor  $42AB - 35A - 30B + 25$

Do this by grouping:

$$\begin{aligned}
 &42AB - 35A - 30B + 25 \\
 &= 7A(6B - 5) - 5(6B - 5) \\
 &= (7A - 5)(6B - 5)
 \end{aligned}$$

5) Use the AC method to factor  $5x^2 - 21x + 4$

We want two numbers whose product is 20 and whose sum is -21: they are -20 and -1. Use them to split the middle term

$$\begin{aligned}
 &5x^2 - 21x + 4 \\
 &= 5x^2 - 20x - 1x + 4
 \end{aligned}$$

Factor by grouping:

$$\begin{aligned}
 &= 5x(x - 4) - 1(x - 4) \\
 &= (5x - 1)(x - 4)
 \end{aligned}$$

6) Factor  $x^2 - 81y^6$

This is a difference of squares:

$$x^2 - 81y^6 = x^2 - (9y^3)^2 = (x - 9y^3)(x + 9y^3)$$

- 7) Solve by using the Zero Product Property:

$$x^2 + x - 20 = 0$$

$$(x + 5)(x - 4) = 0$$

$$\implies x + 5 = 0 \text{ or } x - 4 = 0 \text{ (the Zero Product Property)}$$

$$x + 5 = 0 \implies x = -5$$

$$x - 4 = 0 \implies x = 4$$

So  $x = -5$  or  $x = 4$  (answer)

If you want to write your answer as a solution set, write  $\{-5, 4\}$ , or you can write  $x \in \{-5, 4\}$ . The symbol  $\in$  means that  $x$  is one of the numbers in the set: in more formal language, we say “ $x$  is an element of the set  $\{-5, 4\}$ ”

But whatever you do, make sure you’ve used the notation correctly and never write that  $x$  equals the set, that is WRONG. It’s better to avoid writing in set language if you don’t totally understand what it means.

- 8) Solve by using the Square Root Property:

$$x^2 = 16$$

$$\implies x = \pm\sqrt{16} \text{ (the Square Root Property)}$$

$$x = \pm 4 \text{ (answer), or you can write: } x = 4 \text{ or } x = -4$$

If you want to write your answer as a solution set, write  $\{4, -4\}$ , or you can write  $x \in \{4, -4\}$

- 9) Solve by using the Square Root Property: do not use decimal approximations in your answer.

$$\left(x - \frac{3}{7}\right)^2 = \frac{16}{49}$$

$$\left(x - \frac{3}{7}\right) = \pm\sqrt{\frac{16}{49}} \text{ (the Square Root Property)}$$

$$\left(x - \frac{3}{7}\right) = \pm\frac{4}{7}$$

$$x = \frac{3}{7} \pm \frac{4}{7}$$

Now we have to split into two equations in order to finish simplifying:

$$x = \frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$$

or

$$x = \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

Answer:  $x = 1$  or  $x = -\frac{1}{7}$

If you want to write your answer as a solution set, write  $\{1, -\frac{1}{7}\}$ , or you can write  $x \in \{1, -\frac{1}{7}\}$