

In these problems, do not use decimals in your answers: only use fractions or radicals in fully simplified form. Write complex numbers in the form $a + bi$

- 1a) Find the equation of the line which passes through the points $(-2, 1)$ and $(0, -3)$. Put it in slope-intercept form.

The slope of this line is $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{-3-1}{0-(-2)} = \frac{-4}{2} = -2$

So we know that the equation of the line has the form $y = -2x + b$, and from the information given in the problem the y-intercept is the point $(0, -3)$.

The equation of the line is $y = -2x - 3$

- 1b) Find the equation of the line parallel to the line in part (a), which passes through the point $(1, 1)$. Put it in slope-intercept form.

This line has the same slope as the line in part (a), namely, -2 . So its equation has the form $y = -2x + b$.

This time we have not been given the y-intercept, so we use the fact that $(1, 1)$ is a point on the line to find b :

$$1 = -2(1) + b$$

$$1 = -2 + b$$

$$3 = b$$

So the equation of the line is $y = -2x + 3$

- 1c) Find the equation of the line perpendicular to the line in part (a), which passes through the point $(2, 3)$. Put it in slope-intercept form.

The slope of the perpendicular line is the negative reciprocal of -2 , which is $-\frac{1}{-2} = \frac{1}{2}$

So the equation of this line has the form $y = \frac{1}{2}x + b$, and we use the given point to find b .

$$3 = \frac{1}{2}(2) + b$$

$$3 = 1 + b$$

$$2 = b$$

So the equation of this line is $y = \frac{1}{2}x + 2$

- 2) **Solve the system of equations. Indicate your final answer clearly, and check your answer.**

$$-x + y + 2z = 2$$

$$x + y + 3z = 17$$

$$-3x - y + z = -7$$

There are many ways to solve this system, but in all of the efficient ways we follow the same strategy:

Use any two equations to eliminate one of the variables

Use a different pair of equations to eliminate the same variable

Solve the resulting 2 by 2 system

Substitute back into one of the original equations to find the third variable (the one eliminated in the first step)

Here is one way: it's a good idea to think about why I made the particular choices I did

Use the first and second equations to eliminate x :

$$-x + y + 2z = 2$$

$$x + y + 3z = 17$$

$$2y + 5z = 19$$

Now use the second and third equations to eliminate x again:

$$\begin{array}{r} 3x + 3y + 9z = 51 \\ 3(x + y + 3z) = 3(17) \\ -3x - y + z = -7 \end{array} \left. \vphantom{\begin{array}{r} 3x + 3y + 9z = 51 \\ 3(x + y + 3z) = 3(17) \\ -3x - y + z = -7 \end{array}} \right\} \begin{array}{r} -3x - y + z = -7 \\ \hline 2y + 10z = 44 \end{array}$$

Now we have a 2 by 2 system:

$$2y + 5z = 19$$

$$2y + 10z = 44$$

Subtract the first of those equations from the second to eliminate y :

$$\begin{array}{r} 2y + 10z = 44 \\ -(2y + 5z) = -19 \end{array} \left. \vphantom{\begin{array}{r} 2y + 10z = 44 \\ -(2y + 5z) = -19 \end{array}} \right\} \begin{array}{r} 2y + 10z = 44 \\ -2y - 5z = -19 \\ \hline 5z = 25 \end{array}$$

and we get $5z = 25 \implies z = 5$

Substitute back into one of the equations in the 2 by 2 system to find y : I will use the second equation

$$2y + 10(5) = 44 \implies 2y + 50 = 44 \implies 2y = -6 \implies y = -3$$

Now choose one of the equations in the original 3 by 3 system to find x : I will use the second one

$$x + (-3) + 3(5) = 17$$

$$x - 3 + 15 = 17$$

$$x + 12 = 17$$

$$x = 5$$

The solution to the 3 by 3 system is $(5, -3, 5)$

3) Factor $9x^6 + 12x^8 = 3x^6(3 + 4x^2)$

4) Factor $35AB + 21A + 25B + 15$

Do this by grouping:

$$35AB + 21A + 25B + 15$$

$$= 7A(5B + 3) + 5(5B + 3)$$

$$= (7A + 5)(5B + 3)$$

5) Use the AC method to factor $5x^2 - 19x - 30$

We want two numbers whose product is -150 and whose sum is -19: they are -25 and 6. Use them to split the middle term

$$5x^2 - 19x - 30$$

$$= 5x^2 - 25x + 6x - 30$$

Factor by grouping:

$$= 5x(x - 5) + 6(x - 5)$$

$$= (5x + 6)(x - 5)$$

6) Factor $16x^2 - y^6$

This is a difference of squares:

$$16x^2 - y^6 = (4x)^2 - (y^3)^2 = (4x - y^3)(4x + y^3)$$

7) Solve by using the Zero Product Property:

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$\implies x - 5 = 0 \text{ or } x + 4 = 0 \text{ (the Zero Product Property)}$$

$$x - 5 = 0 \implies x = 5$$

$$x + 4 = 0 \implies x = -4$$

So $x = 5$ or $x = -4$ (answer)

If you want to write your answer as a solution set, write $\{5, -4\}$, or you can write $x \in \{5, -4\}$. The symbol \in means that x is one of the numbers in the set: in more formal language, we say “ x is an element of the set $\{5, -4\}$ ”

But whatever you do, make sure you’ve used the notation correctly and never write that x equals the set, that is WRONG. It’s better to avoid writing in set language if you don’t totally understand what it means.

8) Solve by using the Square Root Property:

$$x^2 = 49$$

$$\implies x = \pm\sqrt{49} \text{ (the Square Root Property)}$$

$$x = \pm 7 \text{ (answer), or you can write: } x = 7 \text{ or } x = -7$$

If you want to write your answer as a solution set, write $\{7, -7\}$, or you can write $x \in \{7, -7\}$

9) Solve by using the Square Root Property: do not use decimal approximations in your answer.

$$\left(x - \frac{3}{7}\right)^2 = \frac{16}{49}$$

$$\left(x - \frac{3}{7}\right) = \pm\sqrt{\frac{16}{49}} \text{ (the Square Root Property)}$$

$$\left(x - \frac{3}{7}\right) = \pm\frac{4}{7}$$

$$x = \frac{3}{7} \pm \frac{4}{7}$$

Now we have to split into two equations in order to finish simplifying:

$$x = \frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$$

or

$$x = \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

Answer: $x = 1$ or $x = -\frac{1}{7}$

If you want to write your answer as a solution set, write $\{1, -\frac{1}{7}\}$, or you can write $x \in \{1, -\frac{1}{7}\}$