Definition; n-th root

For any natural number $n > 1$, we say B is an n-th root of A if and only if $A = B^n$

Note: there are differences in the properties of these roots depending on whether n is even or odd. Pay attention to this.

Notation: $\sqrt[n]{A}$ represents an n-th root of A.

If n is odd, there is only one real-number n-th root of A, and this is it.

If n is even, and $A > 0$, there are two real-number n-th roots of A, and this notation specifically refers to the positive root.

Special case: for the square root (n=2) we do not write a 2 in the radical sign.: $\sqrt{A}$

Products of radicals of the same index n:

$\sqrt[n]{A}\,\sqrt[n]{B} = \sqrt[n]{AB}$

If n is even, both A and B must be ≥ 0

Quotient of radicals of the same index n:

$\frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \sqrt[n]{\frac{A}{B}}$

B must not be 0, and if n is even, both A and B must be ≥ 0

Note: The following two definitions are designed so that the rules about working with exponents above will still work correctly even with rational number exponents. This is (partly) explained by the motivations given after the definitions.

Definition of $\frac{1}{n}$ power:

For any natural number n, $n > 1$,

$A^{1/n} := \sqrt[n]{A}$

Note: If n is even, A must be ≥ 0.

Motivation:

$\sqrt{x^8} = \sqrt{(x^4)} = x^4$ for example, so the square root is acting as if it were a $\frac{1}{2}$ power:

$\left(x^8\right)^{1/2} = x^{8(1/2)} = x^4$

You can try to think of similar examples for the cube root, 4th root, and so on.

Definition of $\frac{m}{n}$ power:

For natural numbers m and n, if $A \geq 0$,

$x^{m/n} := \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$

Note: This definition will work for all real numbers A if both m and n are odd. But be careful!

Motivation:

We can think of $x^{m/n}$ as $(x^m)^{1/n}$ or as $\left(x^{1/n}\right)^m$