Definition; natural number exponents $b^n = \underbrace{b \cdot b \cdot \cdots \cdot b}_{\text{n factors}}$

[Note: $b^1 = b$, only one "factor".]

Product of powers of the same base:

$b^n \cdot b^m = b^{n+m}$

add the powers

Quotient of powers of the same base:

$\frac{b^n}{b^m} = b^{n-m}$

subtract the powers - top minus bottom

Definition of 0 power:

For any real number b, $b \neq 0$,

$b^0 = 1$

Note: 0^0 is undefined; in fact it is indefinite.

Definition of negative integer powers:

For any real number b, $b \neq 0$, and any natural number n,

$b^{-n} = \frac{1}{b^n}$

Note: 0^{-n} is undefined because it would put 0 in the denominator.

Power to a power

$(b^n)^m = b^{nm}$

multiply the powers

Product to a power

$(ab)^n = a^n b^n$

the power distributes over the product

Quotient to a power

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

the power distributes over the quotient

Negative exponents in the denominator

$\frac{1}{b^{-n}} = b^n$

a negative exponent in the denominator means the same as a positive exponent in the numerator.

Note: putting this together with the definition $b^{-n} = \frac{1}{b^n}$ means we can always turn negative exponents into positive ones!

Negative powers and reciprocals

$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

The -1 power is just a way of telling you to take the reciprocal.

Also: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

The $-n$ power is the n-th power of the reciprocal.

Definition of rational number powers with numerator 1:

For any real number b, and any natural number n,
$b^{1/n} = \sqrt[n]{b}$
If n is even we also require that $b \geq 0$ so the root is a real number.

Definition of rational number powers:

For any real number b, and any natural numbers m and n,
$b^{m/n} = \sqrt[n]{b^m}$
or
$b^{m/n} = \left(\sqrt[n]{b}\right)^m$
If n is even we also require that $b \geq 0$ so the root is a real number.

It does not matter which order the root and power are taken in.