Completing the square to put the equation of a parabola into vertex form

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  $y - 5 + 9 = x^2 - 6x + 9$
- Factor the perfect square trinomial, and simplify the left-hand side:
  $y + 4 = (x - 3)^2$
- Finally move the constant term back to the right-hand side:
  $y = (x - 3)^2 - 4$

Not-so-simple Example: when the leading coefficient is not 1

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- We first move the constant term to the other side: $y - 4 = 3x^2 + 9x$
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  $\frac{y}{3} - \frac{4}{3} = \frac{3x^2}{3} + \frac{9x}{3}$
- Simplify:
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  $\frac{y}{3} - \frac{4}{3} + \frac{9}{4} = x^2 + 3x + \frac{9}{4}$
- Factor the perfect square trinomial, and simplify the left-hand side:
  $\frac{y}{3} + \frac{11}{12} = \left(x + \frac{3}{2}\right)^2$

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  $\frac{y}{3} = \left(x + \frac{3}{2}\right)^2 - \frac{11}{12}$

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  $\frac{y}{3} = \left(x + \frac{3}{2}\right)^2 - \frac{11}{12}$
- Multiply both sides by 3 to get y all by itself:
  $3\left(\frac{y}{3}\right) = 3\left(x + \frac{3}{2}\right)^2 - 3\left(\frac{11}{12}\right)$

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