

-

Section 7.1: Completing the square to solve quadratic equations

Square root property: If $x^2 = a$ then $x = \pm\sqrt{a}$ So if $(x + b)^2 = a$ then $x + b = \pm\sqrt{a}$	Perfect square patterns: $(x + b)^2 = x^2 + 2bx + b^2$ $(x - b)^2 = x^2 - 2bx + b^2$
---	---

Examples

1) Solve for x : $x^2 + 6x - 3 = 0$

The left-hand side cannot be factored. We will rewrite this equation so that the left-hand side has the form of a perfect square trinomial:

Move the constant term to the other side (add 3 to both sides) and leave space on the left to add something:

$$x^2 + 6x + (\underbrace{\quad}) = 3 + \underbrace{\quad}$$

Find half of the coefficient of x , and square it: add that square to both sides above.

This makes the left-hand side have the form of a perfect square:

$$(x + \underbrace{\quad})^2 = \underbrace{\quad}$$

Solve using the square root property:

2) Solve for x : $x^2 + 5x - 2 = 0$

The left-hand side cannot be factored. We will rewrite this equation so that the left-hand side has the form of a perfect square trinomial:

Move the constant term to the other side (add 2 to both sides) and leave space on the left to add something:

$$x^2 + 5x + (\underbrace{\quad}) = 2 + \underbrace{\quad}$$

Find half of the coefficient of x , and square it: add that square to both sides above.

This makes the left-hand side have the form of a perfect square:

$$(x + \underbrace{\quad})^2 = \underbrace{\quad}$$

Solve using the square root property:

3) Solve for x : $3x^2 - 8x + 7 = 0$

The left-hand side cannot be factored. We will rewrite this equation so that the left-hand side has the form of a perfect square trinomial:

Move the constant term to the other side (subtract 7 from both sides)

$$3x^2 - 8x + = -7$$

Since the leading coefficient is not 1, we will divide both sides by that number before completing the square:

$$\underbrace{\frac{3x^2}{3}} - \underbrace{\frac{8x}{3}} + = \underbrace{\frac{-7}{3}}$$

Simplify, and leave space to add something on the left-hand side:

Find half of the coefficient of x , and square it: add that square to both sides above.

This makes the left-hand side have the form of a perfect square:

$$(x + \underbrace{\quad})^2 = \underbrace{\quad}$$

Solve using the square root property: