MAT 1375 Final Exam Review Problems

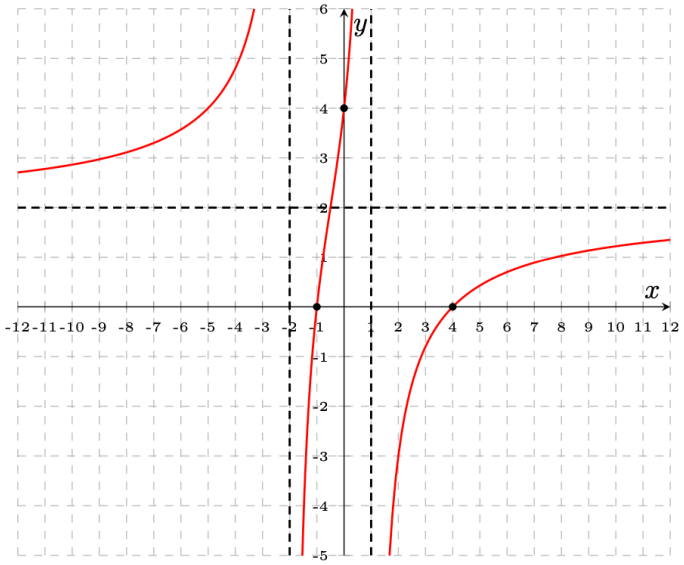
This is a set of review problems to prepare for the MAT 1375 final exam. Solve each problem, show all work, and ensure your answers are written in proper mathematical format.

1. 1. Solve the inequality. Express the solution both on the number line and in interval notation. Use exact forms (such as fractions) instead of decimal approximations.

- a) $x^2 - 3x - 4 \geq 0$
- b) $5x^2 - 7x < 0$
- c) $\frac{(4x-5)}{(2x+3)} \leq 0$
- d) $\left(\frac{3x+4}{7x-9}\right) \geq 0$

2.

(a) A complete graph of the rational function $y = f(x)$ is displayed below. The numerator and denominator of f are polynomials of degree 2, and all asymptotes and intercepts of f are at integer values.



Find all intercepts, asymptotes, and the domain of f. Find a formula for the function f.

(b) A complete graph of the rational function $y = f(x)$ is displayed below. The numerator of f is a polynomial of degree 1, the denominator of f is a polynomial of degree 3, and all asymptotes and intercepts of f are at integer values.

3. Find the difference quotient f(x + h) - f(x)/h (assume h ≠ 0).

- a) $f(x) = x^2 + 4x + 3$
- b) $f(x) = 3x^2 - 2x + 1$
- c) $f(x) = -x^2 + 5x - 6$
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4. For each polynomial, find a real number C so that the polynomial has the indicated root. For this C, find all remaining roots of the polynomial algebraically and write the roots in simplest radical form. Sketch a complete graph of the polynomial, indicating the roots.

- a) $f(x) = x^3 + 3x^2 + C$ has a root at x = -1
- b) $f(x) = x^3 - 2x^2 + C$ has a root at x = 2
- c) $f(x) = 2x^3 + x + C$ has a root at x = -2
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5. Find the magnitude and direction angle.

- a) $v = \langle 3, -3\sqrt{2} \rangle$
- b) $v = \langle -4, -4 \rangle$
- c) $v = \langle -6\sqrt{2}, 6 \rangle$

6. Find the quotient or product and write the result in standard complex form.

(a) $\frac{8(\cos 240^\circ + i \sin 240^\circ)}{4(\cos 120^\circ + i \sin 120^\circ)}$

(b) $3(\cos 135^\circ + i \sin 135^\circ) \cdot 5(\cos 45^\circ + i \sin 45^\circ)$

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7. Let $u = \ln x$ and $v = \ln y$, where $x, y > 0$. Write the following expressions in terms of u and v.

(a) $\ln (x^7 \cdot \sqrt[4]{y})$

(b) $\ln \left(\frac{\sqrt{x^3}}{y^5} \right)$

(c) $\ln (\sqrt[3]{x^2 \cdot y^4})$

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8. Find the domain, asymptotes, and x-intercepts of the function, and then sketch its graph.

(a) $f(x) = \ln(2x - 5)$

(b) $f(x) = \log(9 - 4x)$

(c) $f(x) = \log_3(5x + 4)$

9. State the amplitude, period, and phase shift, and then sketch one complete cycle of the graph. Label all maxima, minima, and x-intercepts.

(a) $y = 4\cos(3x - \frac{\pi}{2})$

(b) $y = 3\sin(5x + \pi)$

(c) $y = -2\sin(x + \frac{\pi}{4})$

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10. Find all exact solutions in radians.

(a) $3\sin^2 x - \sqrt{2}\sin x = 0$

(b) $2\cos^2 x + \cos x - 1 = 0$

(c) $\tan^2 x - 2\tan x = 0$

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11. In 2023, the population of a town is 15,000, and it is decreasing exponentially at a rate of 2% per year.

(a) What will the population be after 7 years?

(b) In what year will the population be half its current size?

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12. In 2020, the population of a city is 100,000 people, and it is growing at a rate of 3% per year.

(a) What will the population be in 2030?

(b) In what year will the population double?

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13. Find the inverse of the following functions.

(a) $y = 5 - 3x$

(b) $y = \frac{6}{2x-1}$

(c) $y = \frac{3x}{4x+7}$

(d) $y = \frac{x+3}{x-2}$

14. Find the sum of the specified terms of the arithmetic sequence.

(a) Sum of the first 60 terms: $15, 11, 7, 3, \ldots$

(b) Sum of the first 100 terms: $-20, -15, -10, -5, \ldots$

(c) Sum of the first 500 terms: $2, 6, 10, 14, \ldots$

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15. Find the exact sum of the infinite geometric sequence.

(a) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

(b) $64, -32, 16, -8, \ldots$

(c) $5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}, \ldots$

(d) $-108, -36, -12, -4, \ldots$

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16. Solve for x.

(a) $4^{2x+1} = 8^{x-2}$

(b) $7^{x+2} = 3^{2x-1}$

(c) $9^{1-x} = 27^{x+3}$
