Rayleigh optical depth comparisons from various sources

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Rayleigh optical depth values obtained from various computations, tabulations, and parameterizations are not always in good agreement. Important differences as large as 3 or 4% can arise depending on the choice of depolarization factor, the formula used for the refractive index of air, and the choice of standard values for columnar and molecular number densities. The fitting equations generally give rise to the largest differences. The use of different standard altitude profiles for atmospheric pressure and temperature causes a variation of 1% or less in Rayleigh optical depth.

I. Rayleigh Optical Depths for the Atmosphere

Solar disk measurements are often used to determine atmospheric extinction by means of the Langley method. More specifically, total optical depths are determined from the slopes of plots in which the logarithms of the solar radiometer voltages are plotted against air mass. The aerosol optical depth at a given wavelength can then be obtained by subtracting the Rayleigh from the total optical depth in spectral regions where absorption is not significant.

Rayleigh scattering from molecules in the atmosphere has been discussed in a wide variety of scientific publications. In a small fraction of these, one can find the basic equations used to compute Rayleigh optical depth, or perhaps a tabulation of Rayleigh optical depths as a function of wavelength, under standard atmospheric pressure and temperature conditions. Occasionally, there will also be a best-fit mathematical equation for easy use in computational codes. Rayleigh optical depth values from these various sources are not always in good agreement.

The basic equation for Rayleigh optical depth is[1]

$$\delta_R = \frac{8\pi^3 (n^2-1)^2 N_c}{3\lambda^4 N_s^2}\left(\frac{6+3\gamma}{6-7\gamma}\right)\left(\frac{p}{p_o}\right)\left(\frac{T_o}{T}\right), \tag{1}$$

where δ_R = Rayleigh optical depth,
n = refractive index of air,
γ = depolarization factor,
N_c = columnar number density (= 2.154×10^{25} cm^{-2} for standard conditions),
N_s = molecular number density (= 2.547×10^{19} cm^{-3} for standard conditions),
λ = wavelength,
p = pressure (= p_o = 1013.25 mb for standard conditions), and
T = temperature (= T_o = 288.15 K for standard conditions).

The pressures p and p_o should be given in consistent units, whereas the temperatures T and T_o must be given in Kelvins. If the wavelength is not given in centimeters, an additional factor must be included (e.g., 10^{16} in the numerator for λ in μm). For the remainder of this discussion, it is assumed that standard atmospheric conditions prevail such that $p = p_o$ and $T = T_o$.

The Elterman tables for Rayleigh optical depths can be found, with slight variations, in a number of places (for example, Elterman,[2,3] Valley,[4] Zuev,[5] Hoyt,[6] Iqbal[7]). Tabulations have also been provided by Penndorf,[1] Coulson,[8] and Frohlich and Shaw.[9]

Numerous fitting equations have been formulated. From Leckner,[10] and as mentioned by Iqbal,[7]

$$\delta_R = 0.008735 \cdot \lambda^{-4.08}. \tag{2}$$

From Moller,[11] and as mentioned by Hill and Sturm,[12]

$$\delta_R = 0.00879 \cdot \lambda^{-4.09}. \tag{3}$$

From Margraff and Griggs,[13] and as mentioned by Pinker and Ewing,[14]

$$\delta_R = 0.0088 \cdot \lambda^{(-4.15+0.2\cdot\lambda)}. \tag{4}$$

Frohlich and Shaw[9] generated a fit of their computational results:

$$\delta_R = 0.00838 \cdot \lambda^{\alpha}, \tag{5}$$

where

$$\alpha = -3.916 - 0.074 \cdot \lambda - 0.05/\lambda.$$

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From the LOWTRAN-5 code,[15] and as mentioned by Bird and Riordan,[16]

$$\delta_R = \lambda^{-4} \cdot (115.6406 - 1.3366 \cdot \lambda^{-2})^{-1}. \tag{6}$$

From Hansen and Travis,[17] and as mentioned by Gordon *et al.*,[18]

$$\delta_R = 0.008569 \cdot \lambda^{-4}(1 + 0.0113 \cdot \lambda^{-2} + 0.00013 \cdot \lambda^{-4}). \tag{7}$$

In all of these cases, it is assumed that the wavelength λ is in μm.

II. Depolarization Factor

The choice of depolarization factor γ is one of the chief reasons why published Rayleigh optical depths are not always in agreement. An excellent discussion of this problem may be found in a series of articles by Young.[19–21] The standard tables of Penndorf[1] and Elterman[3] from many years ago used the value $\gamma = 0.035$. Some years later, Hoyt[6] published revised Rayleigh optical depth values based on $\gamma = 0.0139$, but excluding the effect of Raman lines. As Young pointed out, however, Raman scattering is part of molecular scattering and the full depolarization effect must be used to compute optical depths. Unfortunately, Hoyt's incorrect value of γ was adopted by others,[9,22] including the developers of the 5S atmospheric code from France[23] and a Discrete-Ordinate-Method code from Canada.[24] Based on more recent depolarization data for dry air, Young recommends a value of $\gamma = 0.0279$.

III. Refractive Index of Air

A widely used formula for the index of refraction of dry air is that of Edlen[25]:

$$10^8(n-1) = 8342.13 + \frac{2406030}{130 - \lambda^{-2}} + \frac{15997}{38.9 - \lambda^{-2}}, \tag{8}$$

where n is the refractive index, λ is the wavelength in μm, and standard atmospheric conditions are assumed (pressure = 1013.25 mb, temperature = 288.15 K). Written in this form, Eq. (8) is referred to as a dispersion formula. Edlen's 1966 formula is an improved version of an earlier equation[26]:

$$10^8(n-1) = 6432.8 + \frac{2949810}{146 - \lambda^{-2}} + \frac{25540}{41 - \lambda^{-2}}. \tag{9}$$

In the 0.3–3.5 μm wavelength range, the differences between n values from Eq. (8) and Eq. (9) are very small.

Fenn *et al.*[27] represent Eq. (8) in a different but mathematically equivalent expression:

$$10^6(n-1) = 83.42 + \frac{185.08}{1 - \left(\frac{1}{11.40\lambda}\right)^2} + \frac{4.11}{1 - \left(\frac{1}{6.24\lambda}\right)^2}. \tag{10}$$

Another formula that matches Edlen[25] very closely is that of Peck and Reeder[28]:

$$10^8(n-1) = \frac{5791817}{238.0185 - \lambda^{-2}} + \frac{167909}{57.362 - \lambda^{-2}}. \tag{11}$$

Apparently this equation generates slightly better values than Edlen's 1966 formula, compared to experimental data in the IR. Peck and Reeder also point out that four coefficients are sufficient, five parameters being necessary only for wavelengths 0.23 μm.

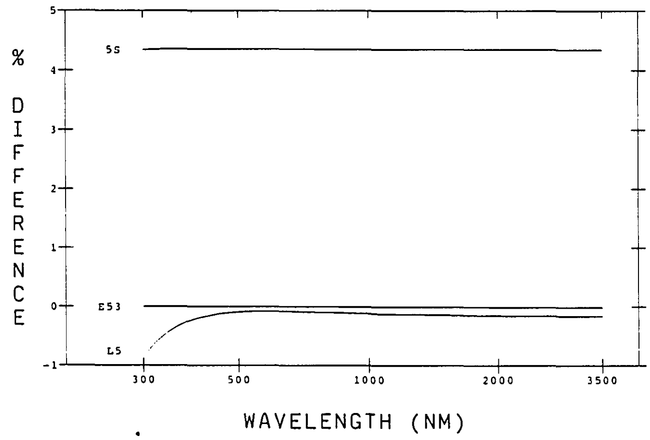

Fig. 1. Percentage difference in the factor $(n^2 - 1)^2$ as a function of wavelength, where n is the refractive index of air. Factors based on formulas for n due to Edlen[26] (E53), LOWTRAN 5[15] (L5), and the 5S code[23] (5S) are compared to the reference case from Edlen.[25]

The LOWTRAN-6 radiative transfer code[29] uses the 1966 Edlen expression for air refractive index, but LOWTRAN-5[15] used the following formula:

$$10^6(n-1) = \left(77.46 + \frac{0.459}{\lambda^{-2}}\right)\left(\frac{1013.25}{288.15}\right). \tag{12}$$

The 5S atmospheric code[23] has adopted yet another formula:

$$10^8(n-1) = 6593.1 + \frac{3010189.3}{146 - \lambda^{-2}} + \frac{26113.82}{41 - \lambda^{-2}}. \tag{13}$$

A comparison of the values for n obtained from all of the above formulas in the 0.3–3.5-μm wavelength range shows very little difference between them, the largest difference being 0.0006% between 5S and Edlen 1966. However, Rayleigh optical depth computations depend on $(n^2 - 1)^2$. Figure 1 illustrates the percentage differences between $(n^2 - 1)^2$ values based on the various formulas, using Edlen 1966 as the reference case. Results from the 5S expression differ by 4.4%. For the Peck and Reeder values, the differences are on the order of 0.001% or less and, therefore, are not included in the figure.

IV. Comparisons of Rayleigh Optical Depths

To compare different tabulations and equations for Rayleigh optical depth, Eq. (1) was taken as a reference, with $\gamma = 0.0279$, $N_c = 2.154 \times 10^{25}$ cm^{-2}, $N_s = 2.547 \times 10^{19}$ cm^{-3}, $p = p_o$, $T = T_o$, and n from Edlen's 1966 formula. Figure 2 shows the percentage difference from the reference case as a function of wavelength, whereas Fig. 3 plots absolute difference in Rayleigh optical depth from the reference case as a function of wavelength. The fitting equations generally give rise to the largest differences, although the equation of Hansen and Travis[17] is the best match with

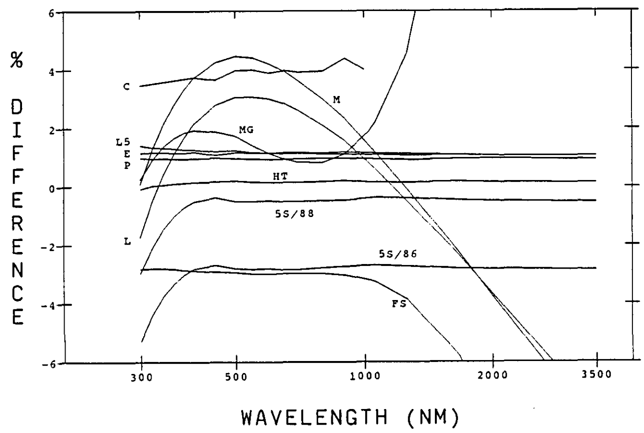

Fig. 2. Percentage difference in Rayleigh optical depths derived from various sources as a function of wavelength. The comparisons are with respect to values from Eq. (1). The different curves are distinguished by the label nearest to the short wavelength end of each curve, where the labels are defined as follows: C = Coulson;[8] L5 = LOWTRAN 5;[15] E = Elterman;[3] P = Penndorf;[1] MG = Margraff and Griggs;[13] M = Moller;[11] HT = Hansen and Travis;[17] L = Leckner;[10] FS = Frohlich and Shaw;[9] 5S/88 = modified 5S code[30] with γ = 0.0279, 5S/86 = 5S code[23] with γ = 0.0139.

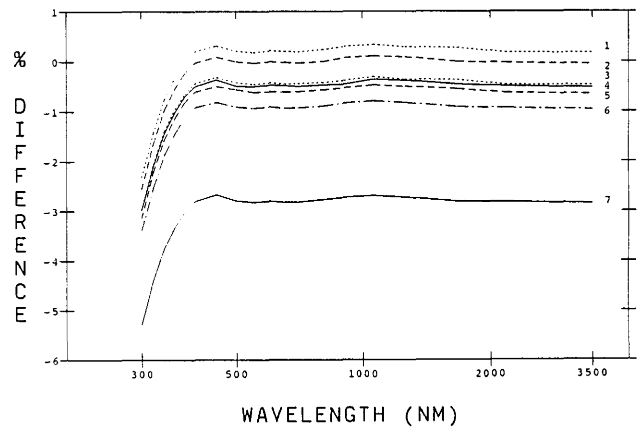

Fig. 4. Percentage difference in Rayleigh optical depths computed by the 5S code for a variety of standard altitude profiles for atmospheric pressure and temperature. The comparisons are between values from Eq. (1). Curve 7 is for a mid-latitude summer profile model and a depolarization factor of γ = 0.0139. The remaining curves are for γ = 0.0279 and the profile models are identified as follows: 1 = mid-latitude winter, 2 = sub-arctic winter, 3 = tropical, 4 = mid-latitude summer, 5 = U.S. Standard Atmosphere 1962, 6 = sub-arctic summer.

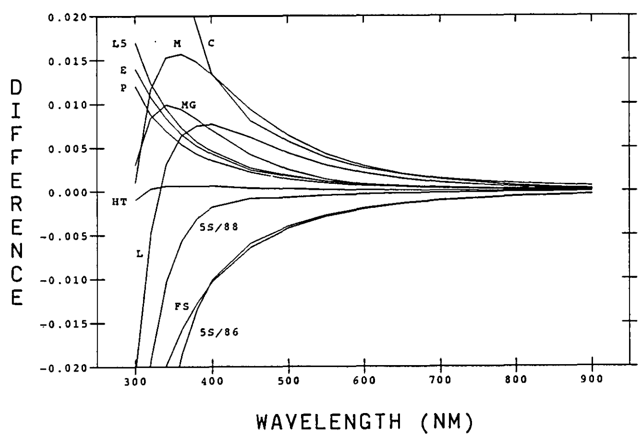

Fig. 3. Absolute difference in Rayleigh optical depths derived from various sources as a function of wavelength. The comparisons are with respect to values from Eq. (1). The different curves are labeled as in Fig. 2.

the reference case. These equations necessarily depend on the source data used in the fitting, but it is evident that the simpler power-law parameterizations are generally not very satisfactory. The choice of depolarization factor causes values from Elterman[3] and Penndorf,[1] for example, to be high and values from Frohlich and Shaw[9] and the 5S code[23] to be low.

Results from the 5S code are based on computations using an equation similar to Eq. (1), but exercised only at certain discrete wavelengths, with interpolations filling in the gaps. Because the lowest wavelength used for an actual computation is 400 nm, Rayleigh optical depths for shorter wavelengths are obtained by extrapolation and begin to deviate quite markedly from the reference values (Figs. 2 and 3). Changing the depolarization factor to 0.0279 in the 5S code[30] results in the curve labeled 5S/88 in Figs. 2 and 3. As noted earlier, the air refractive index formula used in 5S will increase Rayleigh optical depths by 4.4% compared to the reference case. However, the N_c and N_s values chosen for the 5S code are such that this increase is more than completely negated. Figure 4 illustrates the effect of different atmospheric pressure and temperature profiles as a function of altitude on the Rayleigh optical depths obtained from the 5S code. Variations due to the different profiles encompass a ~1% range in Rayleigh optical depth.

V. Concluding Remarks

Rayleigh optical depth values obtained from various computations, tabulations, and parameterizations have been compared. Important differences as large as 3 or 4% can arise depending on the choice of depolarization factor, the formula used for the refractive index of air, and the choice of standard values for columnar and molecular number densities. Fitting equations based on simple power-law representations are generally not as effective as more elaborate parameterization formulas. Results from calculations using a variety of standard altitude profiles for atmospheric pressure and temperature vary by 1% or less.

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