Remote Sensing, EET 3132
Lecture 2

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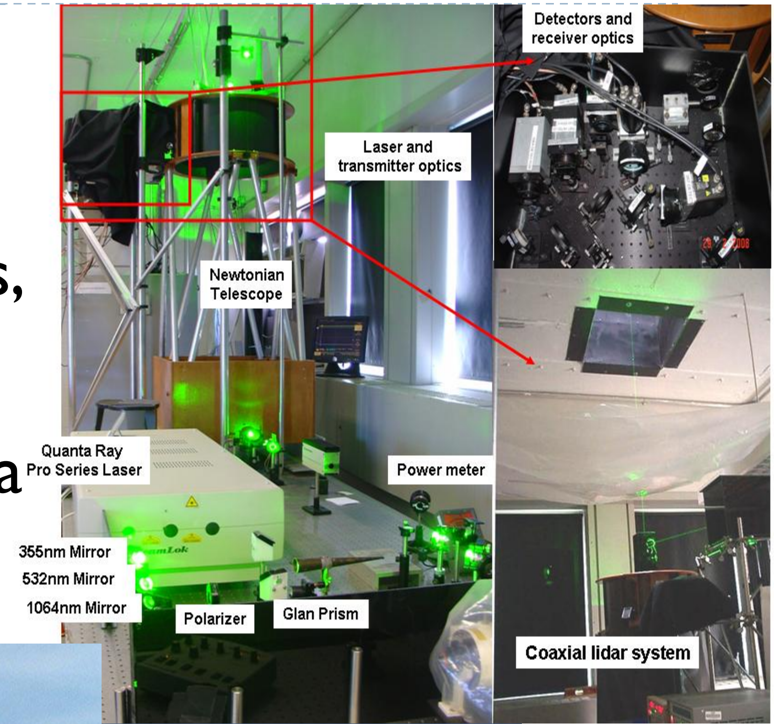
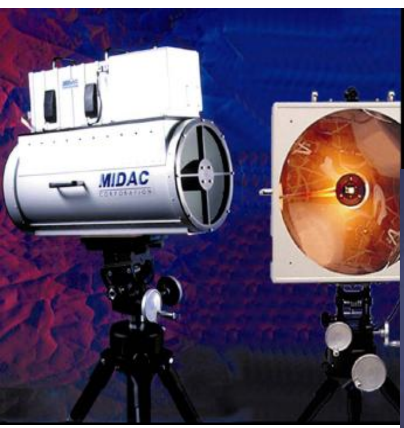
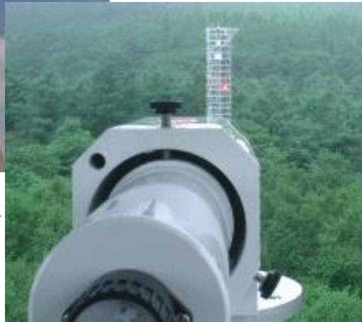
Remote Sensors- Land based

- lidar, radar, sodar, FTIR, sunphotometer, scintillometers, temperature profilers, spectrometers, wind profiler, sky radiometer, microwave radiometer, photo-camera Wireless sensors, etc

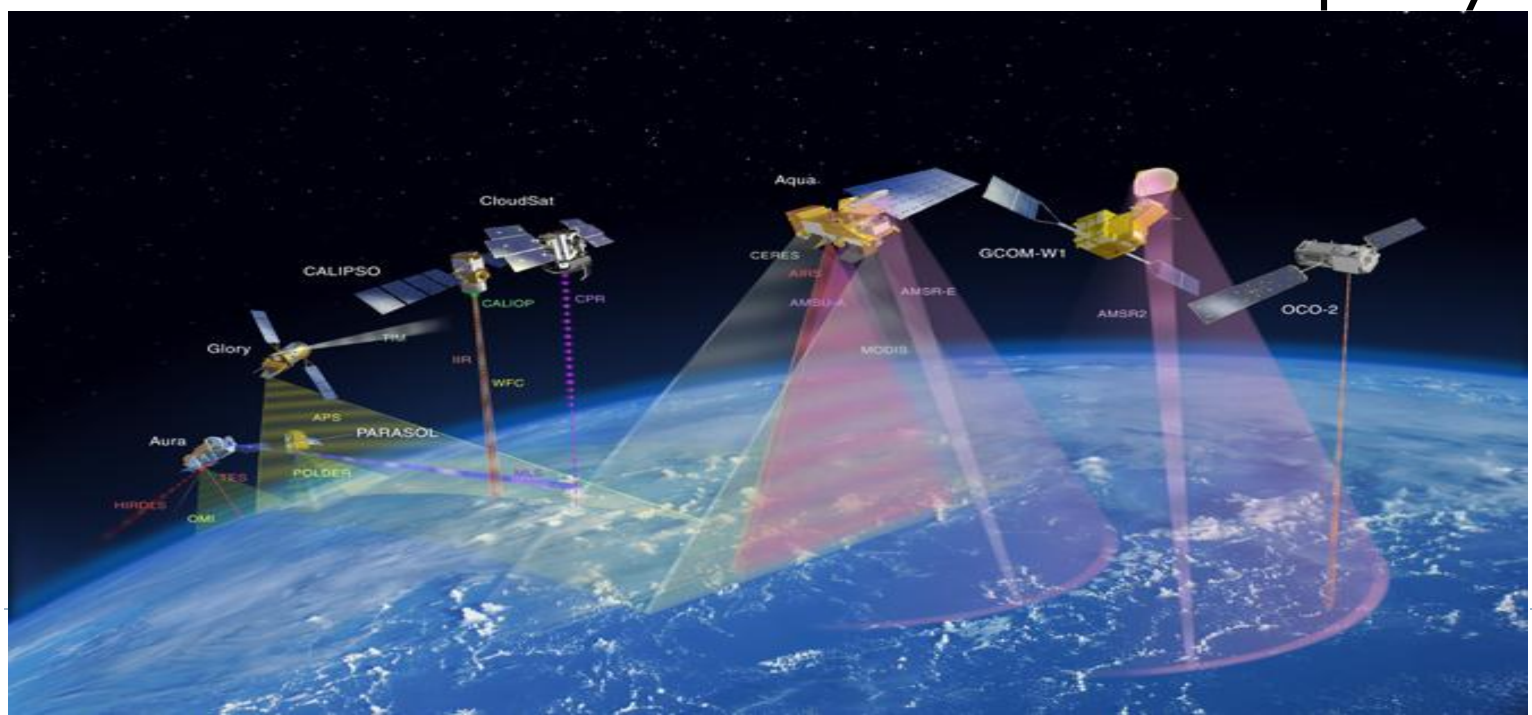




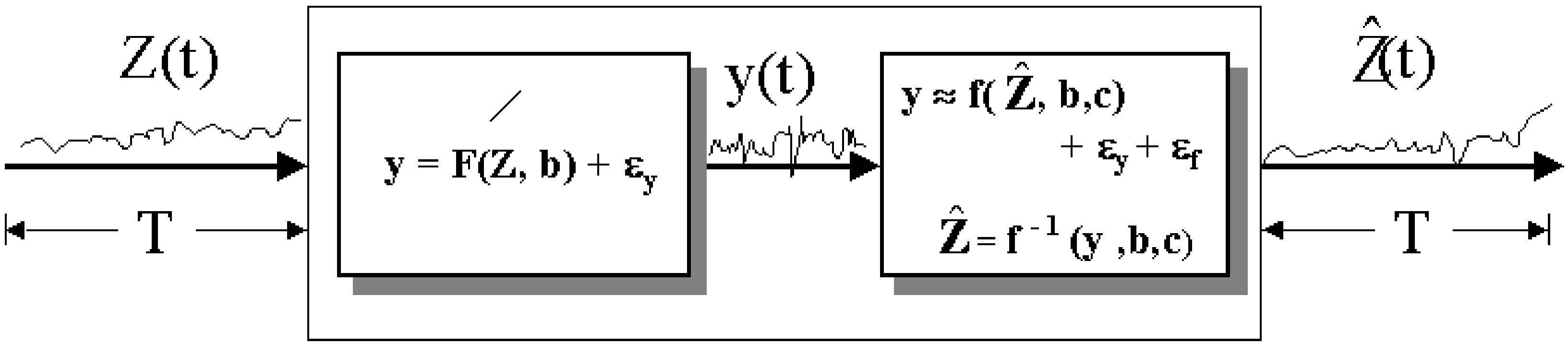


Remote Sensors-Space based

- Remote Sensors in space are installed on board of satellites that combine telescopes for optical information in conjunction with optical trains and high spectral resolution optical sensors like interferometer and spectrometers, charge coupled devices; polarimeters; and other systems. Antennae are used for detection in the radio frequency.



The Observing System Transfer Function

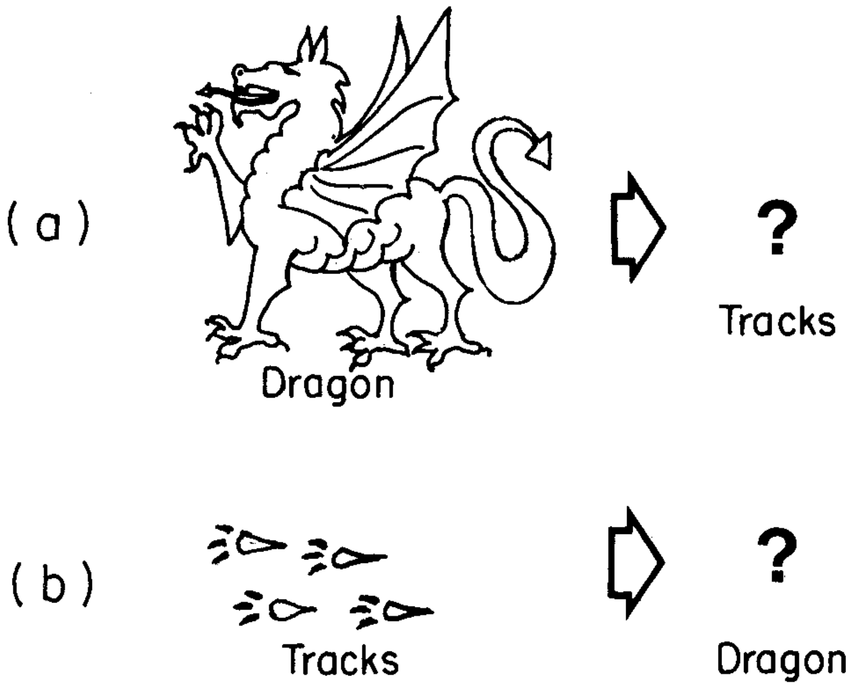


A remote sensing observing system presented in the form of a transfer function (outer box) and component transfer functions (the two inner boxes). This system produces an output Ž that differs from the input Z due to the internal parameters of the observing system. One of the goals of this system is to reconstruct the time variability of Z from the time variability of Ž

Key parameters & steps :

- Measurement, y(t) and error ε_y
- Model f & its error ε_f
- Model parameters b and errors
- Constraint parameters c

The retrieval problem



Forward Problem (real)

$y=F(x) + \varepsilon_y$

y = measurement

F = Nature's forward model

x = parameter desired

ε_y = error in measurement (noise, calibration error,…)

Often the relation between the measurement y and the parameter of interest x is not entirely understood

$$y=f(\hat{X},b) + \varepsilon_y + \varepsilon_f$$

b = 'model' parameters that facilitate evaluation of f

ε_f = error of model

Inverse Problem

$$\hat{x} = I(y,b)$$

The problem is the performance of the ‘system’ is affected by the performance of the individual parts. Examples of issues:

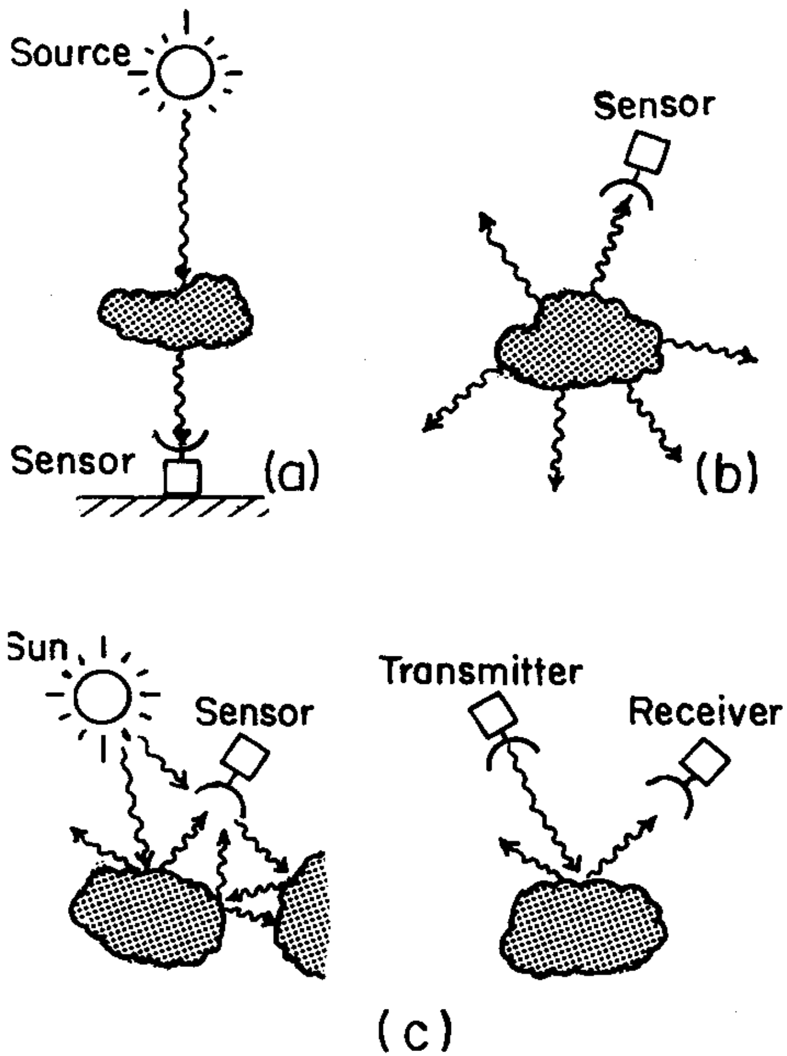
(i) Properly formed forward models – [e.g. Z-R relationships, poorly formed forward model without an understanding of what establishes the links between the observable y(Z) and the retrieved parameter X(R)]

(ii) Need for constraints – refer temperature inversion problem

(iii) Information content – need to know what part of the system dominates retrievals, is it y, is if f or b, is it the constraint c? This controls the error

Really need to understand all of these connected facets to understand the design of the system and the quality of the information that is produced

Experimental Design



Based on some sort of relation defined by a physical process:

(a) extinction - turbidity, SAGE (Stratospheric Aerosol and Gas Experiment)occultation, ..

(b) emission - atmospheric sounding, precipitation,..

(c) scattering - passive, cloud aerosol, ozone,..
- active, radar & lidar

There is a significant progress in designing & using systems that combine different types of observations (and physics) – such as active with passive

The forward problem applied

$y = f(\hat{x},\hat{b}) + \varepsilon_y + \varepsilon_f$

f = our depiction of the forward model

$\hat{x},\hat{b}$ = estimates of x,y

ε_f = F(x,b) - f $(\hat{x},\hat{b})$ + $\partial f/\partial b(\hat{b}-b)$
= error in forward model

Radiative transfer model (most common)

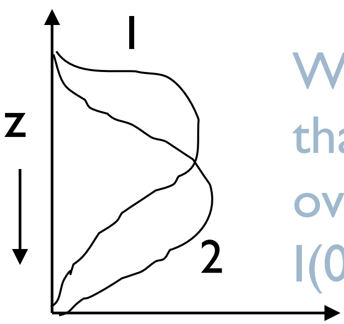
Radiation + physical model

Radiation model + NWP (radiance assimilation)

For the most challenging problems we encounter, it is generally true that the largest uncertainty arise from forward model errors. If you see error estimates on products that exclude these errors – then you ought to be suspicious – really suspicious

Non –uniqueness and instability: Example from emissions

Physically:



Weighting functions that substantially overlap

$I(0) = \int B(z')W(0,z')dz'$

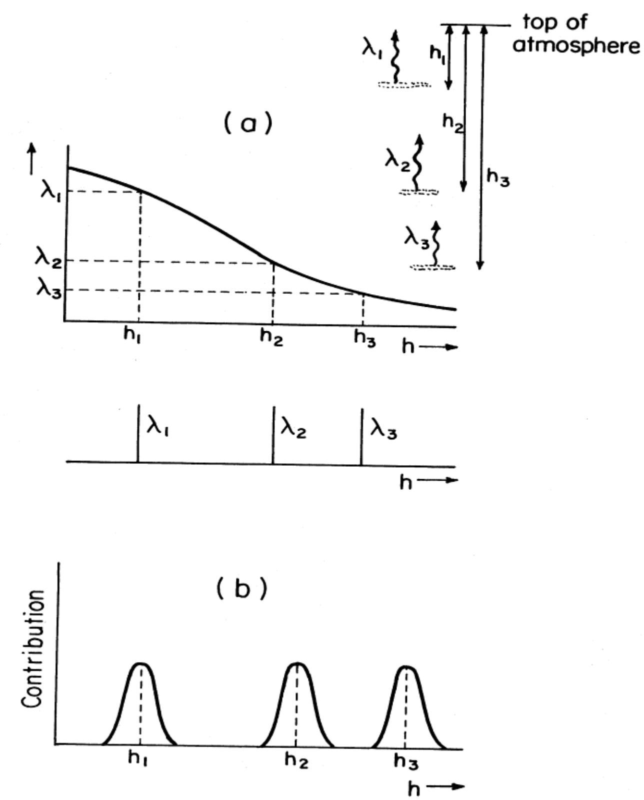


Figure 1.3 (a) Schematic diagram for a satellite-based atmospheric temperature profile measurement. The contribution to the radiation measured at the selected wavelength arises from a discrete level. (b) A version of the contribution function of Fig. 1.3a corresponding to a more practical case where the measured radiation originates from a range of different levels in unequal proportions (from Twomey, 1977).

Mathematically:

$I_1=B_1W_{11}+B_2W_{12}$

$I_2=B_1W_{21}+B_2W_{22}$

suppose:

$W_{11}=W_{12}=1$

$W_{21}=2, W_{22}=2.000001$

$I_1=2$

$I_2=4.000001$ $\quad$ $I_2=4.000000$

then

$B_1=1$ $\quad$ $B_1=2$

$B_2=1$ $\quad$ $B_2=0$

Non-uniqueness and instability Estimation

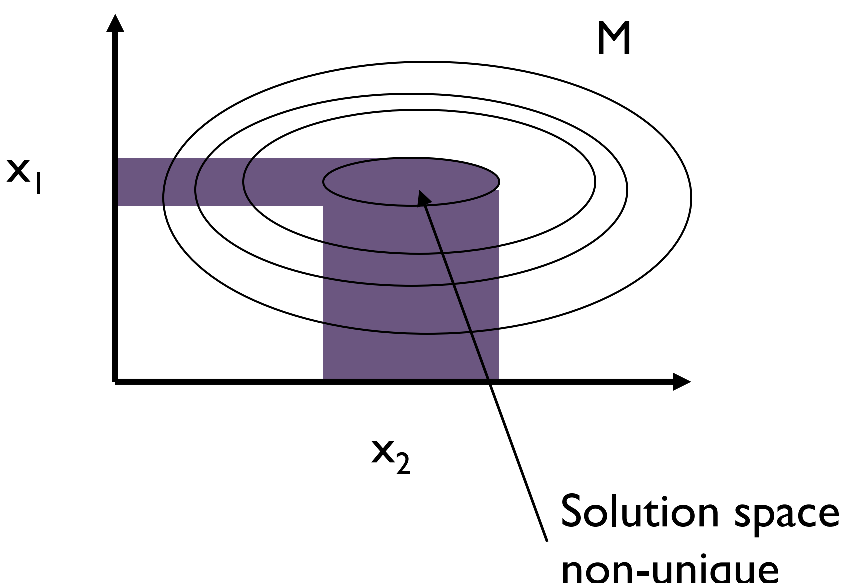
'metric' of length (e.g. least squares)

Cost Function: Φ = M [y-f(x)]

measurement

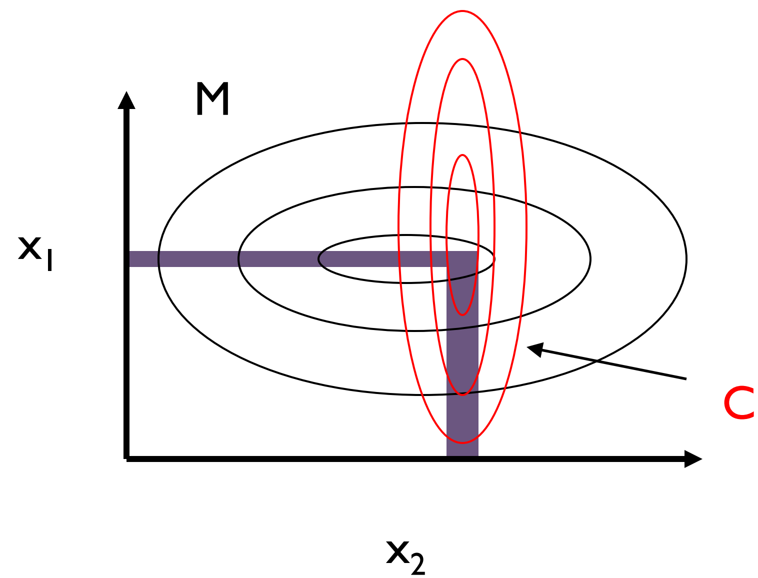
Prediction of measurement

Unconstrained



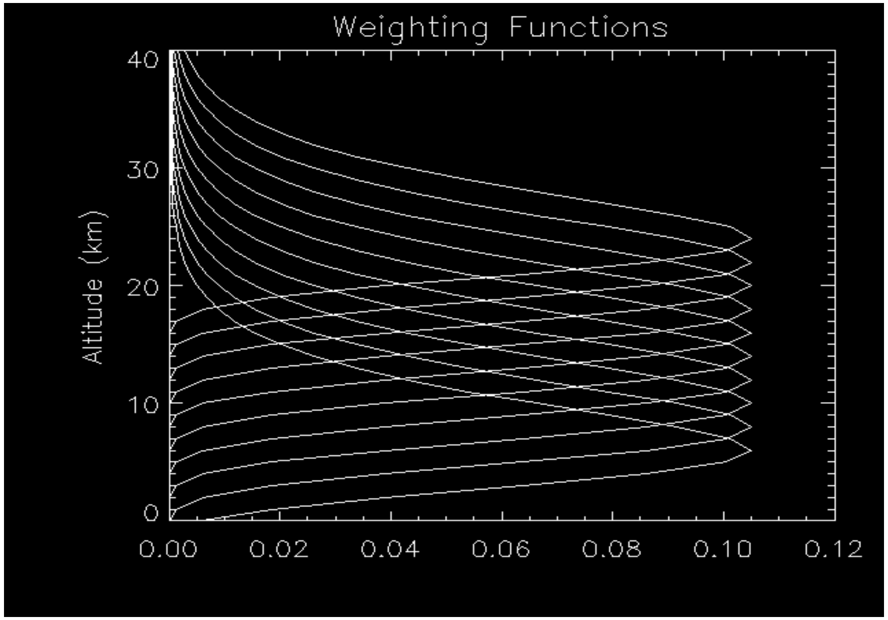
Constrained

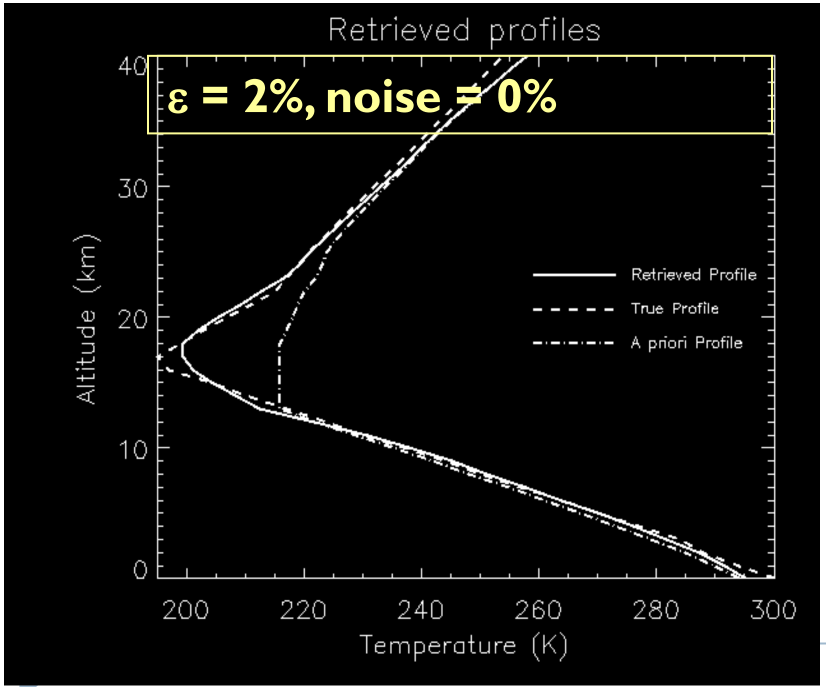
Φ = M [y-f(x)] + C(x)

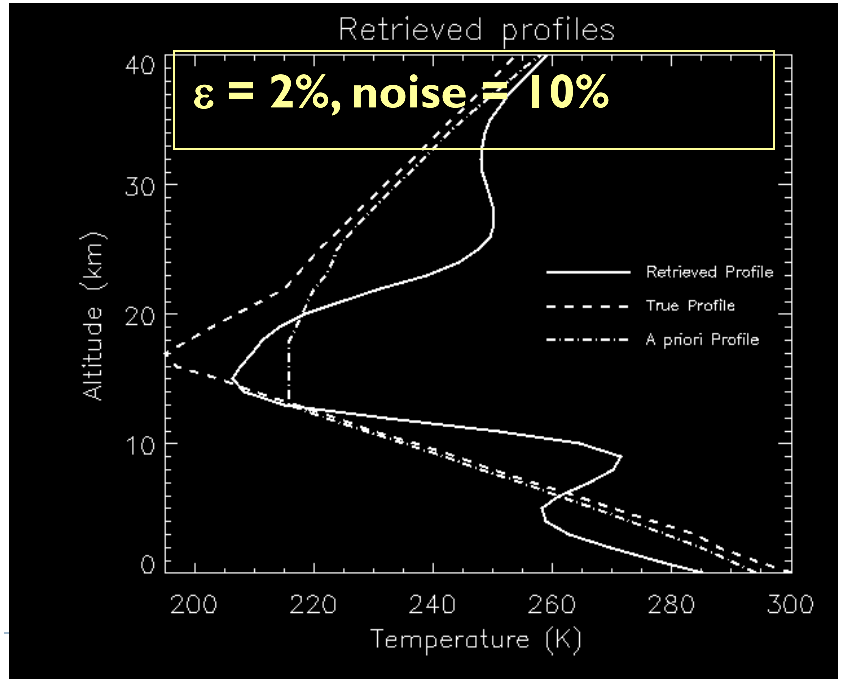


C(x)= initial or *a priori* constraint

Results from temperature retrieval project

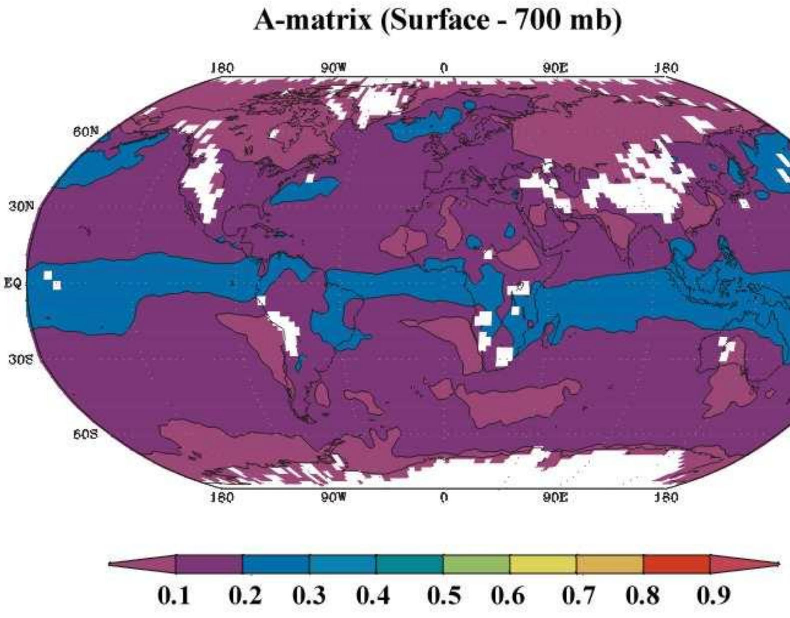


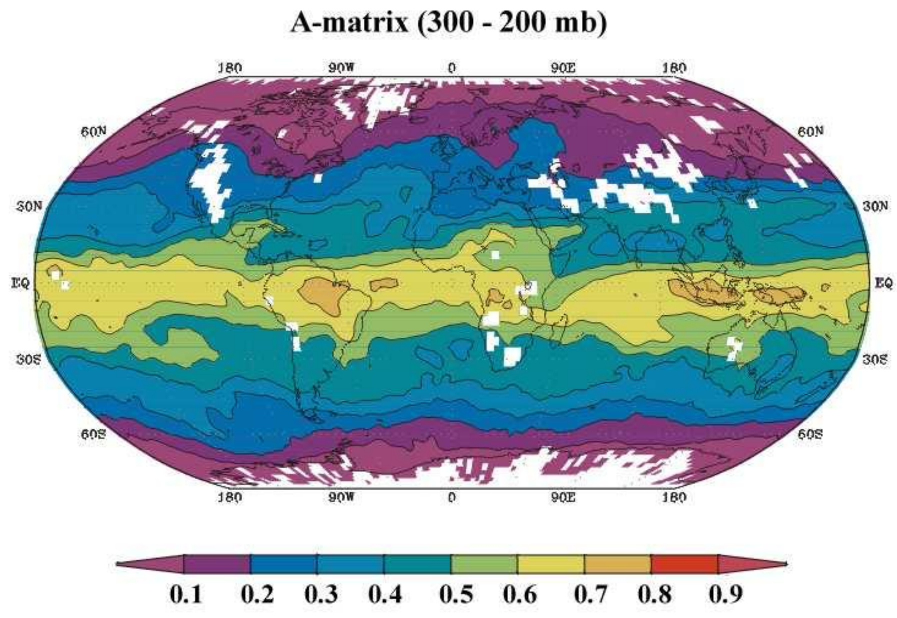




Information Content: The example of IR-based retrieval of water vapor

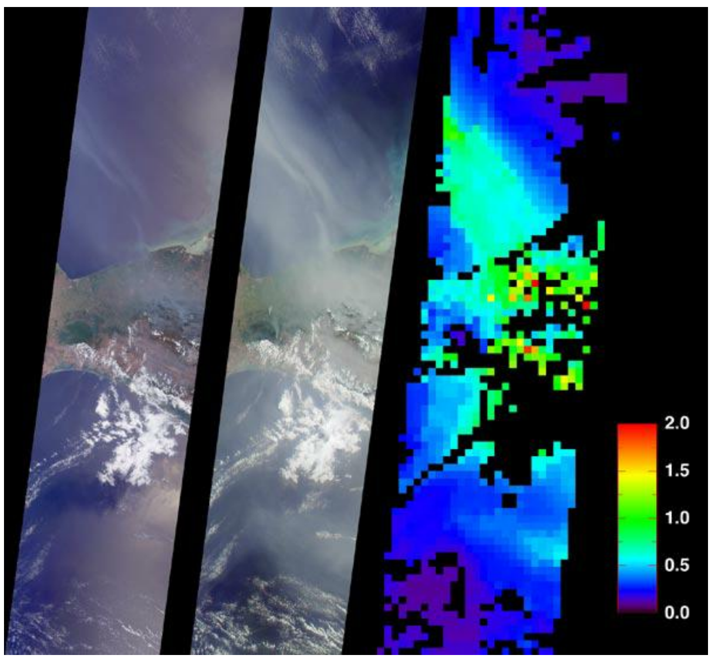
Metric of how much a priori constraint contribute to the retrieval
A→0, all a priori, no measurement
A→1, no a priori, all measurement

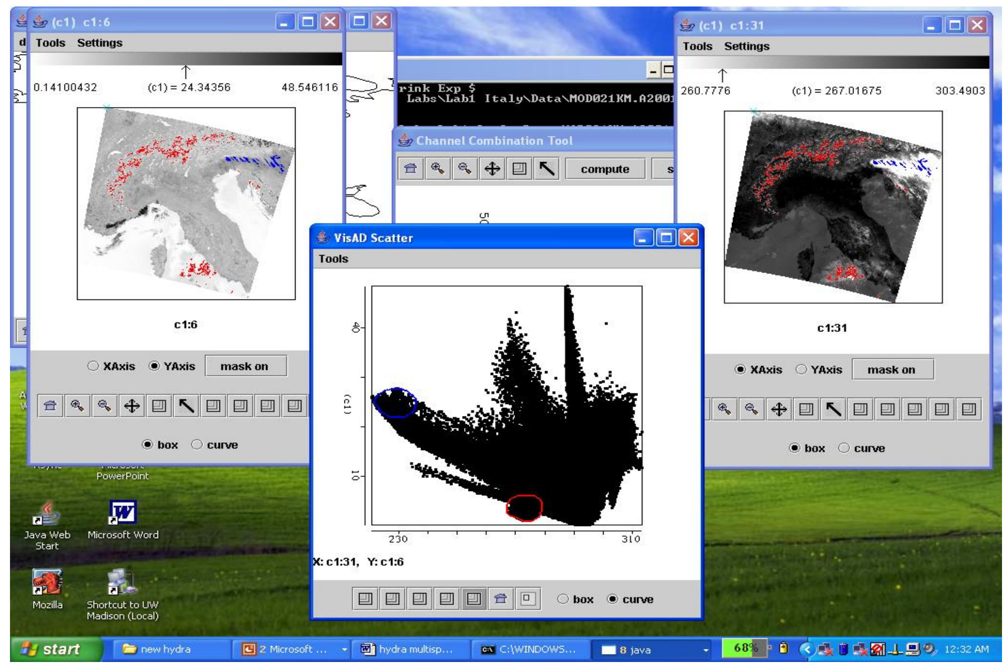




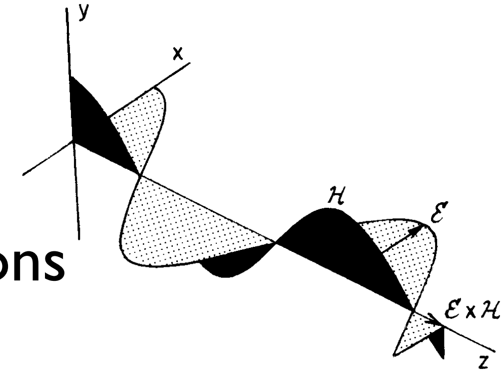
What we measure and how we measure it?

- Energy in the electromagnetic waves
- Polarization (to be discussed in the next lecture)





Electromagnetic waves

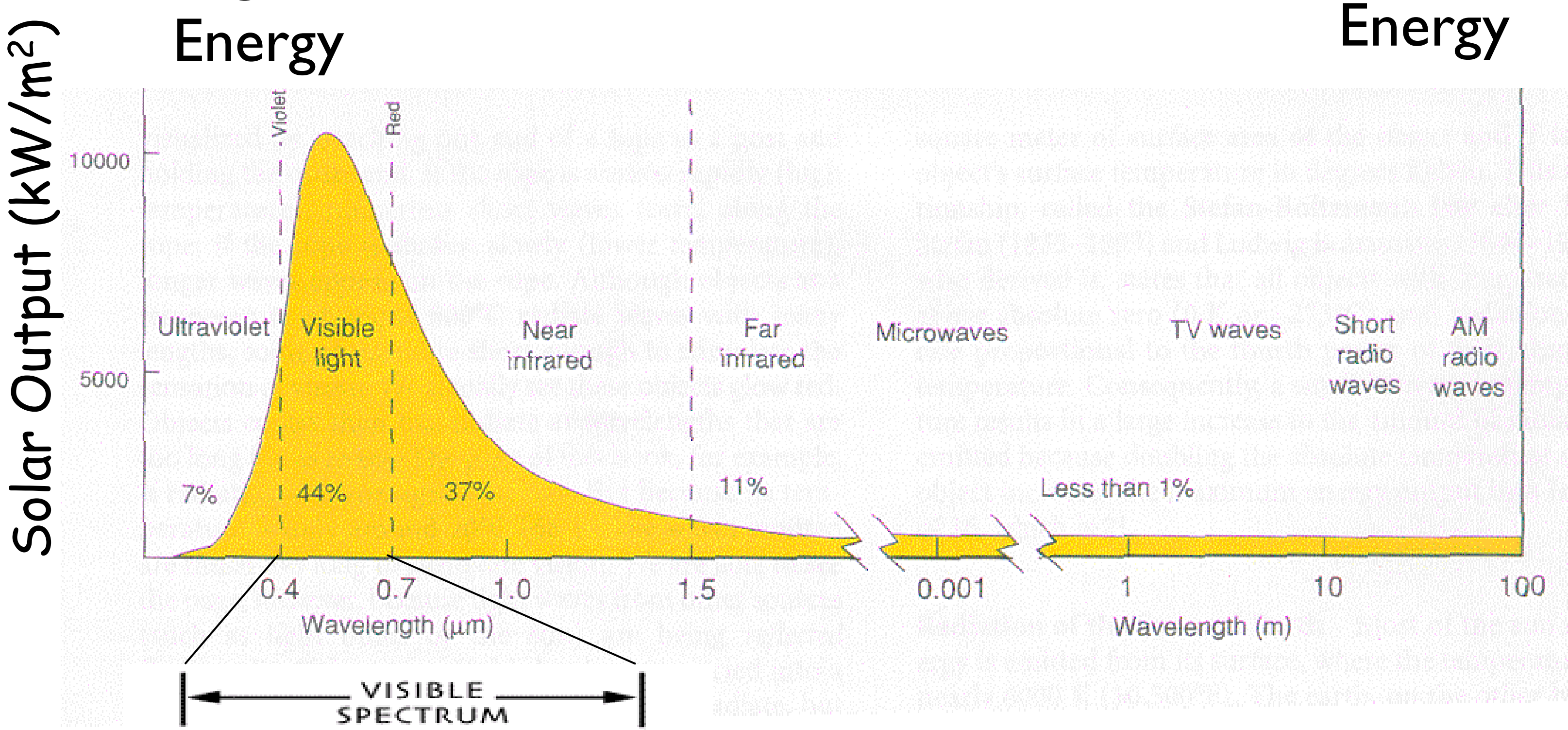


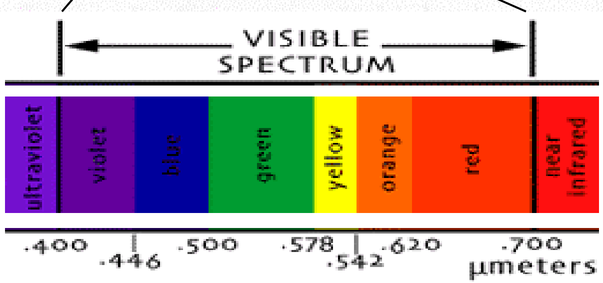
- EM radiation created via the mutual oscillations of the electric ε and magnetic fields *H*
- Direction of propagation is orthogonal to direction of Oscillations
- Speed of travel $c = c_0/n$, where n is the refractive index
- Oscillations described in terms of
  - wavelength λ (distance between individual peaks in the oscillation-
    - e.g. λ=0.7 μm, frequency ν (= c/ λ; the number of oscillations that occur within a fixed (1 sec) period of time - e.g. λ=0.7 μm $\rightarrow$ 4.3 x 10^{14} cycles per second or Hz.
  - wavenumber (= 1/ λ; the number of wave crests (or troughs) counted within a fixed length (say of 1 cm) -
    - e.g. λ=0.7 μm $\rightarrow$ 14286 cm^{-1}

The EM Spectrum

Highest Energy

Lowest Energy

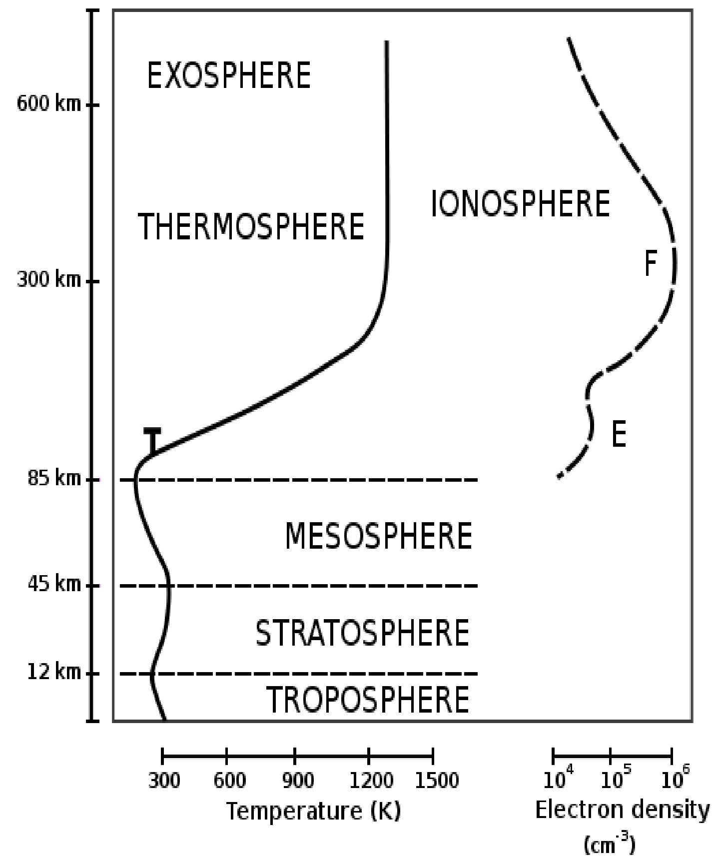




There is little absorption in the visible spectrum (with the exception of carbon)

We are interested in those portions of the EM spectrum wherein most of the energy resides. For solar and terrestrial emissions, this is principally at wavelengths longer than about 0.3 µm and shorter than about 100 µm

Interaction of waves with matter

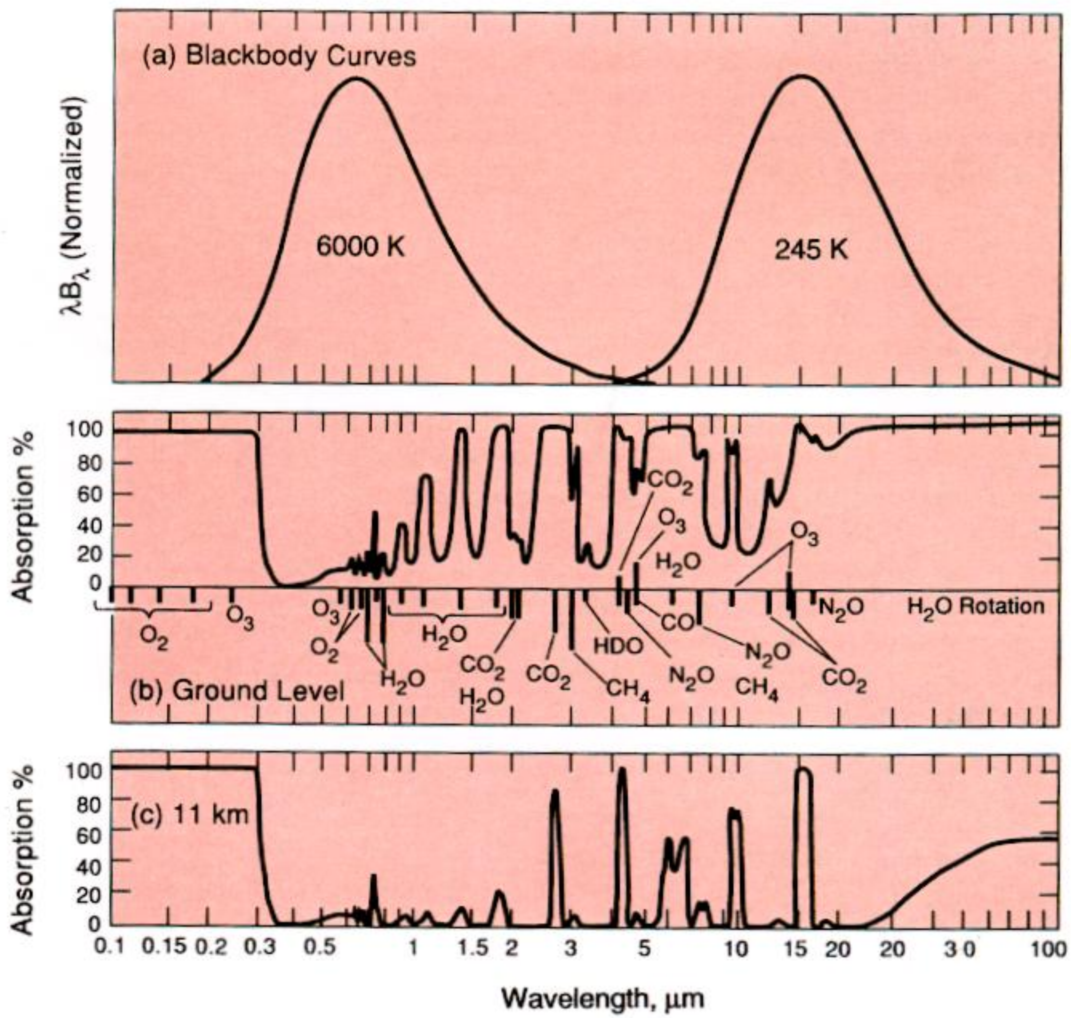


UV Radiation Effects in the lower Ionosphere. EUV ionization of oxygen and nitrogen, release photons upon return to ground state.

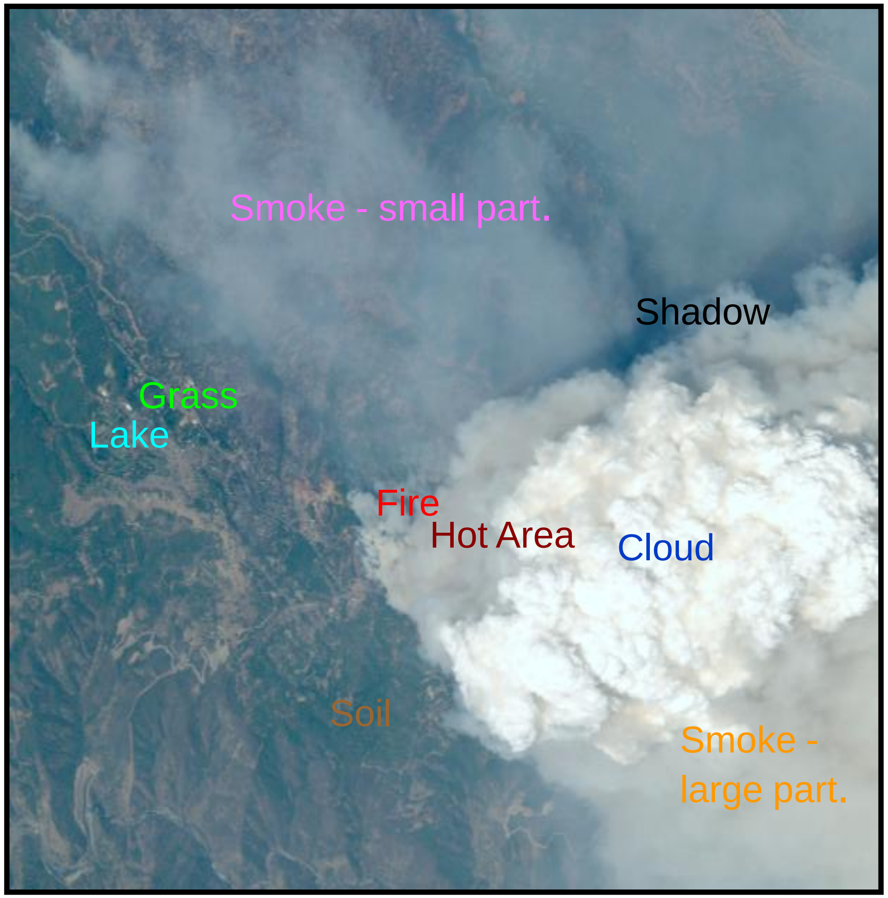
At Earth the (net) solar energy received (~0.3-4 μm) ~ the radiation emitted from the Earth (~>4 μm)

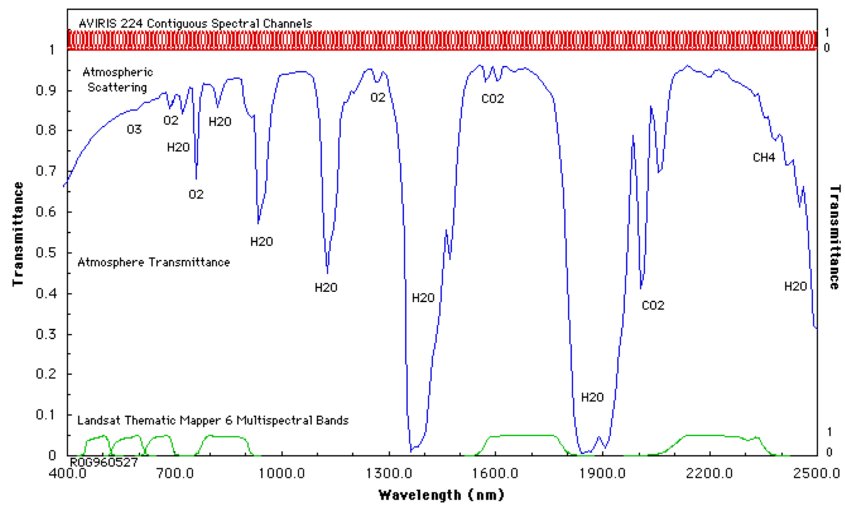
@ Surface

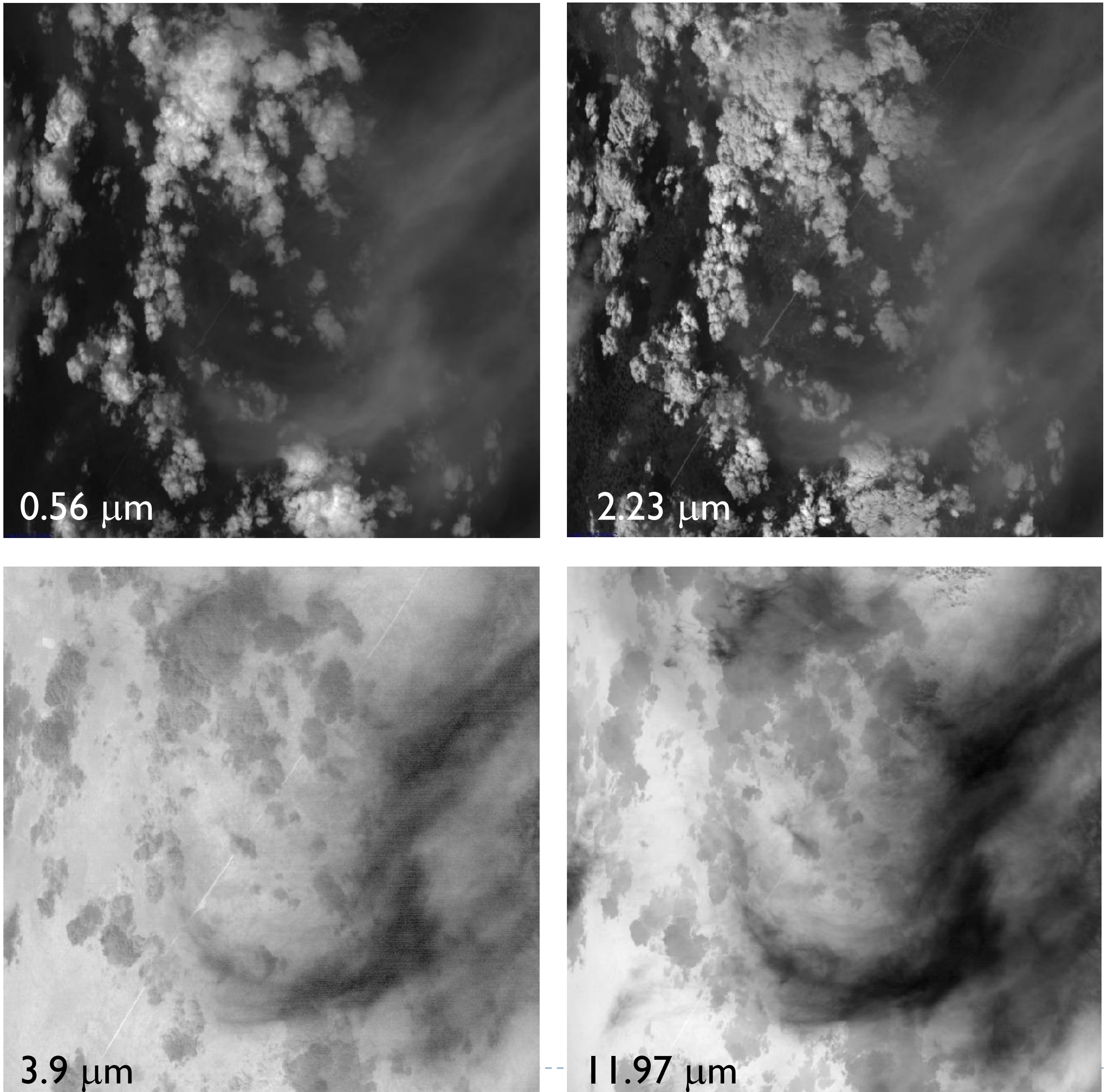
@ Tropopause



Different spectral regions provide characteristic signatures of the Earth and the Atmosphere



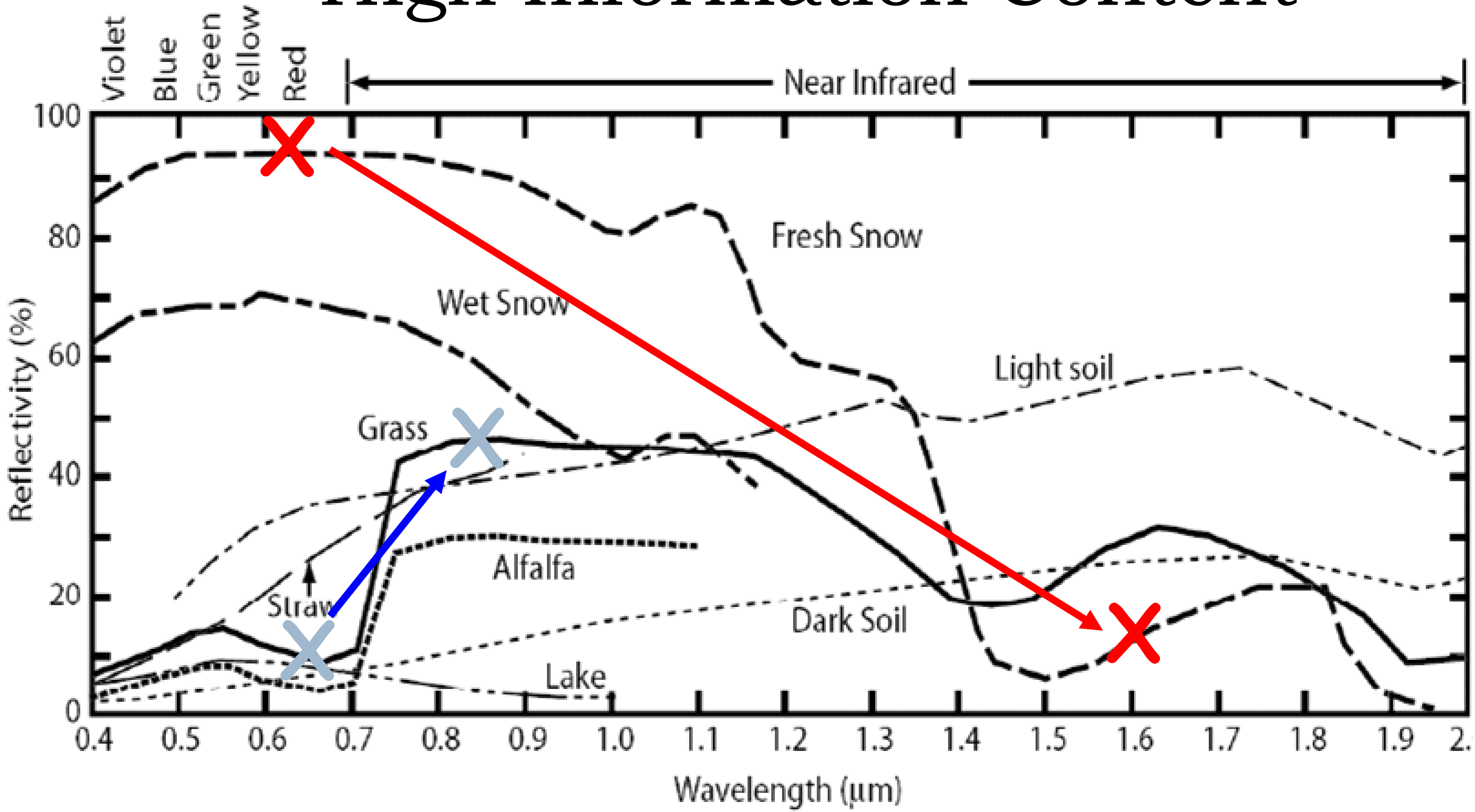




MODIS Airborne Simulator

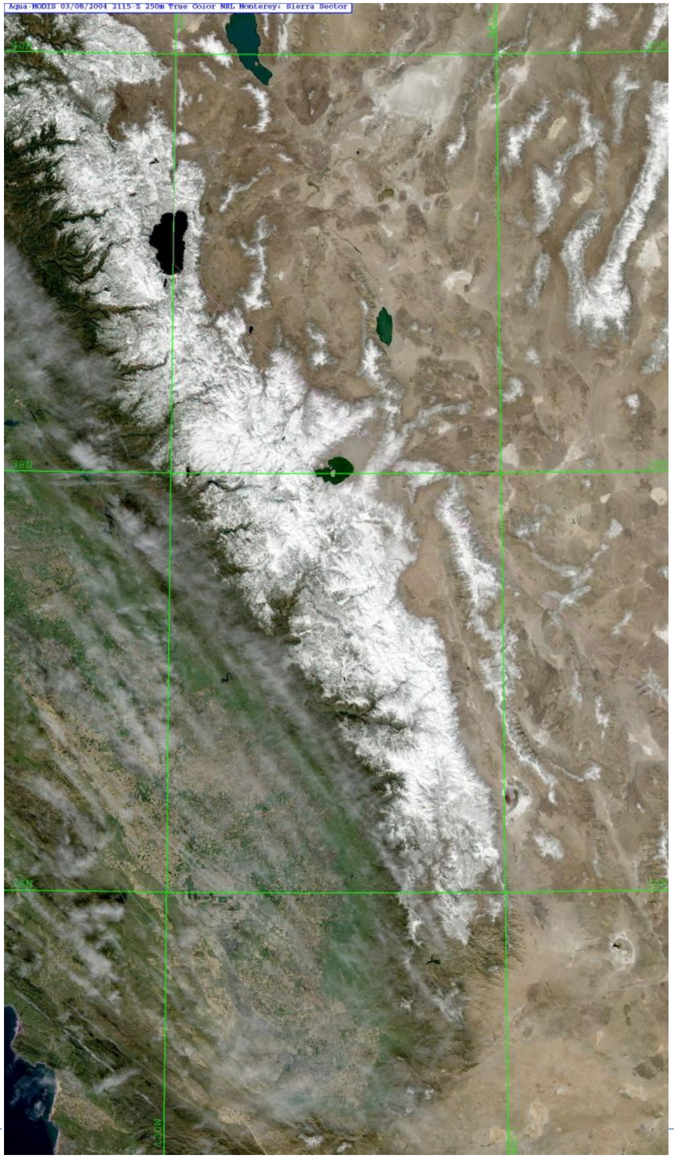
Simultaneous radiances at four channels during CRYSTAL-FACE(A measurement campaign designed to investigate tropical cirrus cloud physical properties and formation processes)

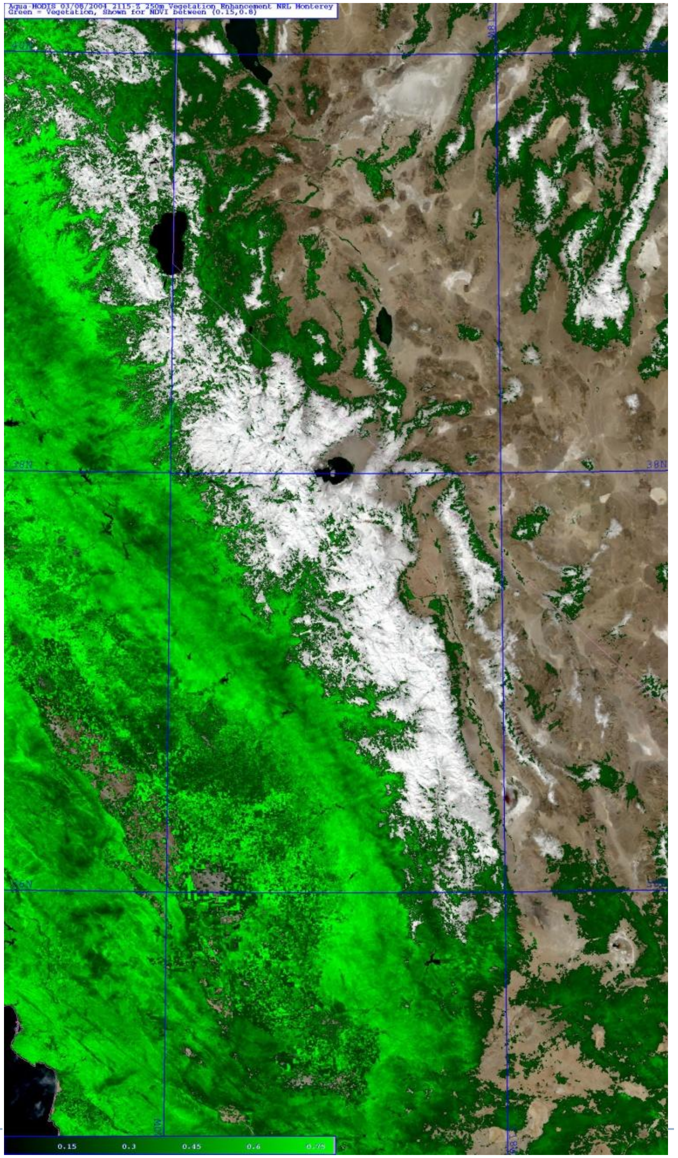
The EM Spectrum Holds High Information Content



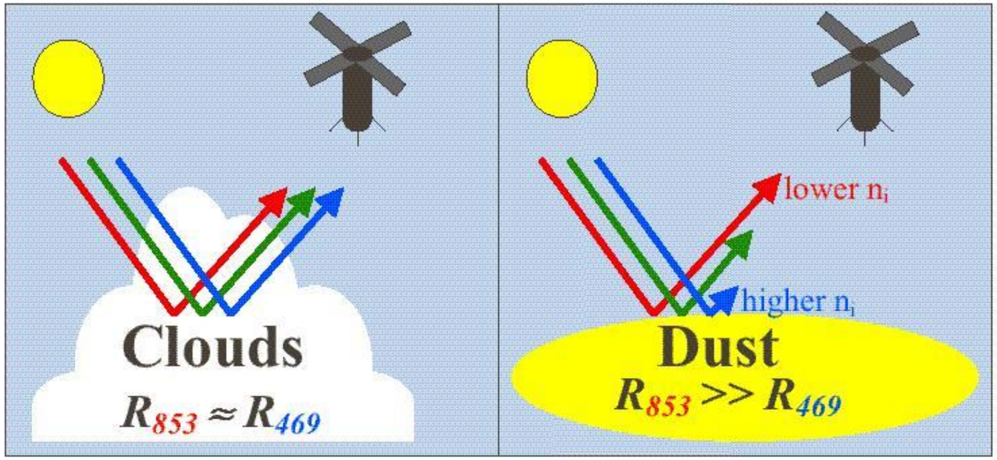
A few remote sensing examples follow…

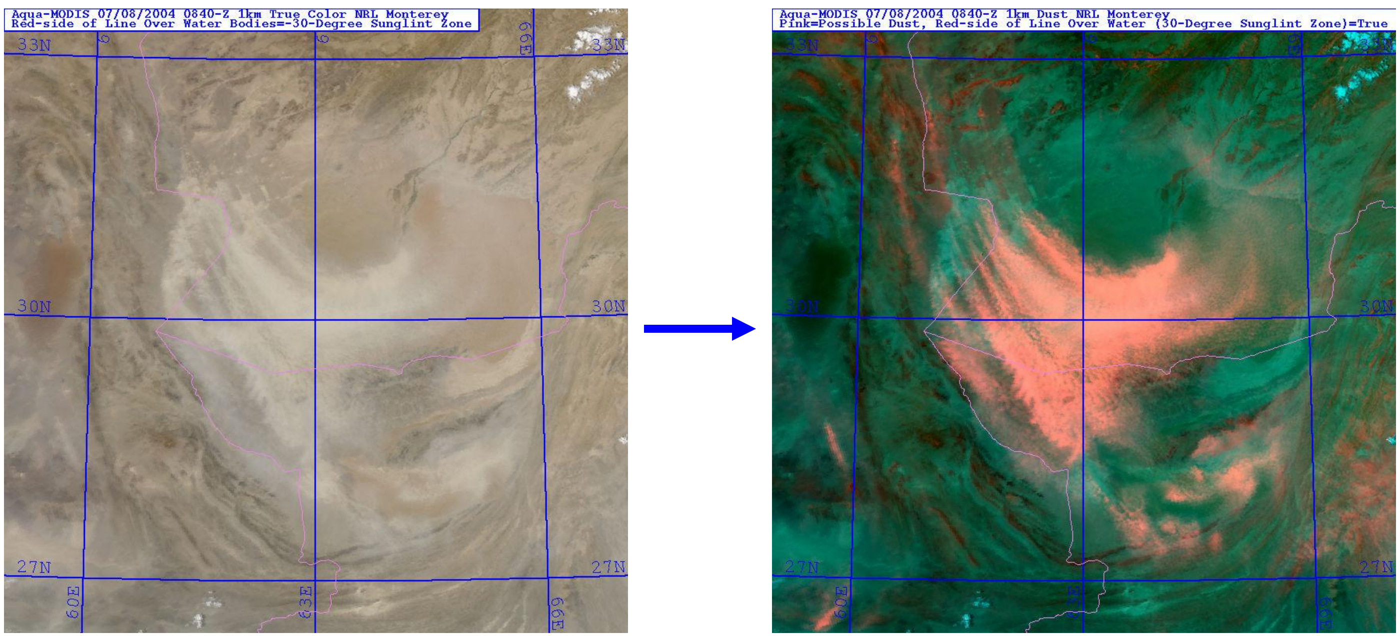
1) Characterizing Vegetation via NDVI = $\dfrac{R(NIR) - R(VIS)}{R(NIR) + R(VIS)}$





Interaction of waves with matter





Blue Light Absorption to Identify Mineral Dust Plumes

Some Useful Terms & Units

- Momentum = mass * velocity = kg m/s
- Force = mass * accel = kg * (m/s)/s = kg m/s^2 (Newtons)
- Pressure = Force/Area = kg / (m s^2) (Pascals)
- Work or Energy = Force * Distance = kg m^2 s^2 (Joules)
- Power = Work/Time = kg m^2 s^3 (Watts)

Several important radiation terms used in this course will follow from these.

(!) With all the terminology and different ways of measuring and representing EM energy (especially considering non-standardized nomenclature), it is always good practice to check your units as a baseline sanity check.

Properties of EM radiation

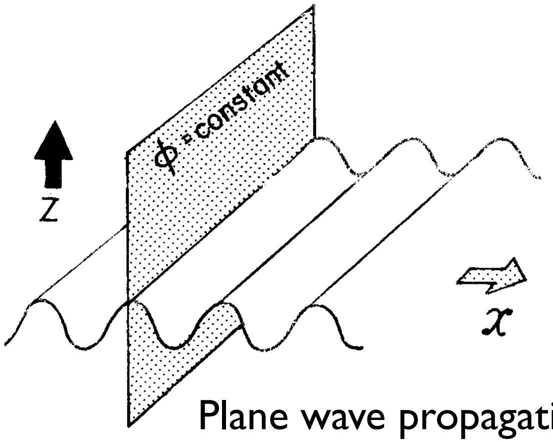
Three basic properties define the EM radiation

- the frequency of oscillation -Rate of oscillation (i.e frequency or wavelength) is very important - it determines how radiation interacts with matter - rule of thumb:
  - fastest oscillations (say UV wavelengths) affect lightest matter (e.g. electrons);
  - slow oscillations (IR and microwave affect larger more massive parts of matter (molecules,….)
- Amplitude of the oscillation ε_o - this directly defines the amount of energy (and entropy)
  - carried by EM radiation. The energy carried is proportional to $|\varepsilon_o|^2$
- Polarization of radiation- this defines the 'sense' of the oscillation - it does not affect the energy carried but it can affect the way radiation interacts with matter.

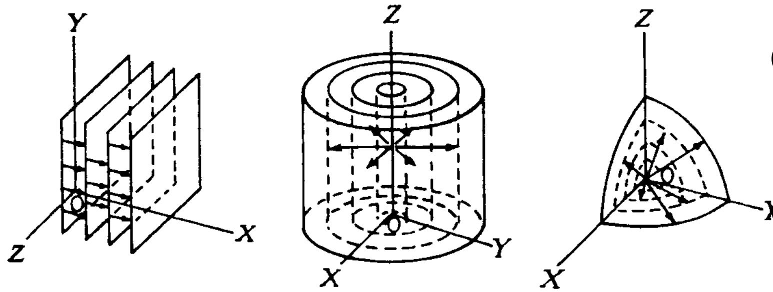
Energy in Electromagnetic waves

- Electromagnetic Radiation:
- The fundamental property is the spectral radiance: which relates to the fundamental properties of an EM- wave - and it is these properties and how they change via interaction with the atmosphere, land and ocean that carry information.
- The properties include: amplitude (energy) and phase (motion) of wave, its spectral dependence (type of interaction that occurs), its direction of propagation and the transverse properties of the wave (polarization).
- Ex. The amount of energy emitted by any body is related to its temperature, via the Planck blackbody relation (to be covered shortly)

Mathematical description of EM radiation



Plane wave propagation along x with plane of constant phase highlighted



Wave propagation in 3D - examples of plane, cylindrical and spherical wave propagation

Plane wave form is expressed as

$\varepsilon(x,t) = \varepsilon_o \cos k(x\text{-}ct) = \cos(kx\text{-}\omega t)$

k=wavenumber (not to be confused with $\tilde{\nu}$)

$k = 2\pi/\lambda$

$\omega = kc$ =angular frequency

$\phi = k(x\text{-}ct) = (kx\text{-}\omega t)$ = phase

or as $\varepsilon(x,t) = \varepsilon_o \exp(i\,\phi)$

Main point: energy carried by the wave, related to $|\varepsilon(x,t)|^2 \rightarrow |\varepsilon_o|^2$, does not get altered by the simple act of propagation (must interact with matter for the energy to be altered

Energy in EM waves

- The wave theory (and light) explains a great deal about phenomena we observe in physics, but at the turn of the 20th century, it became obvious that a different perspective was needed to explain some of the interactions of light and matter- in particular, processes such as the photoelectric effect (and similar processes important for the detection of light). The inadequacy of the wave theory led to a resurgence of the idea that light, or electromagnetic radiation, might better be thought of as particles, dubbed photons. The energy of a photon is given by

$$E = hf$$

- Where f=frequency of the EM wave in Hz and h is the Plank's Constant= $6.626*10^{-34}$ J*s
  = $4.136*10^{-15}$ eV*s (1eV=$1.602*10^{-19}$ Joules)

- *Q? How much energy has a wave in the visible spectrum?*

Energy Carried by the EM Wave: the Poynting Vector

The energy per unit area per unit time flowing perpendicular into a surface in free space is given by the *Poynting vector* $\vec{\mathcal{S}}$, where

$$\vec{\mathcal{S}} = c^2\epsilon_o\vec{\mathcal{E}} \times \vec{\mathcal{H}}, \qquad Wm^{-2}$$

where c is the speed of light and ϵ_o is the vacuum permittivity.

$$\vec{\mathcal{E}} = \vec{\mathcal{E}}_o \cos(kx - \omega t).$$

$$\vec{\mathcal{H}} = \vec{\mathcal{H}}_o \cos(kx - \omega t)$$

$$\mathcal{S} = c^2\epsilon_o\vec{\mathcal{E}} \times \vec{\mathcal{H}} = c^2\epsilon_o\vec{\mathcal{E}}_o \times \vec{\mathcal{H}}_o \cos^2(kx - \omega t).$$

Hence

$$< \mathcal{S} >= c^2\epsilon_o \mid \vec{\mathcal{E}}_o \times \vec{\mathcal{H}}_o \mid < \cos^2(kx - \omega t) >,$$

and the time average is calculated for an interval of length T according to

$$< \cos^2(kx - \omega t) >= \frac{1}{T}\int_t^{t+T} \cos^2(kx - \omega t')dt'$$

$$= \frac{1}{2} - \frac{1}{4\omega T}[\sin(2kx - 2\omega(t+T)) - \sin 2(kx - \omega t)].$$

When $T \gg t$, $\omega T \gg 1$ and $< \cos^2(kx - \omega t) > \to 1/2$. Since $\mathcal{E}_o = c\mathcal{H}_o$,

$$F =< \mathcal{S} > \approx \frac{c\epsilon_o}{2}\mathcal{E}_o^2$$

or

$$F \approx c\epsilon_o < \mathcal{E}^2 >,$$

where $< \mathcal{E}^2 >= \mathcal{E}_o/2$. It also follows that $I =< S > /d\Omega$

Example 1.2: Consider the following problem: a plane, sinusoidal, linearly polarized electromagnetic wave of wavelength $\lambda = 5.0 \times 10^{-7}$m travels in a vacuum along the x axis. The average flux of the wave per unit area is 0.1 $\mathrm{Wm}^{-2}$and the plane of vibration of the electric field is parallel to the y axis. Write the equations describing the electric and magnetic fields of the wave.

The solution is as follows: The wavenumber is $k = 2\pi/\lambda = 4\pi$ m^{-1} Given u the following,

$$c\epsilon_o = \frac{10^7}{4\pi c}$$

then the amplitude

$$\mathcal{E}_o = \sqrt{\frac{2 \times 0.1}{c\epsilon_o}} = \sqrt{(24\pi)}$$

and the form of the E wave is

$$\mathcal{E}_y(t) = \sqrt{24\pi}\cos 4\pi(x - ct) \quad \mathrm{NC}^{-1}$$

and the magnetic field is governed by

$$\mathcal{H}(t) = \frac{\mathcal{E}_y(t)}{c} \quad \mathrm{T}$$

Main point: energy carried $\rightarrow$ $|\varepsilon_o|^2$

Radiant Flux - A Basic Quantity*

* but one that is not directly measurable

The Radiant Flux (P) along $\vec{\mathbf{r}}$ for monochromatic radiation (i.e., per unit spectral interval, e.g., per micrometer (μm)) is given by:

Photon speed: $c = \lambda\nu$ (m/s)

$$\mathbf{P}(\vec{\mathbf{r}}) = \mathbf{h\nu} \times \mathbf{n}(\vec{\mathbf{r}}) \times \mathbf{c} \times \mathbf{dA}$$

Photon Energy (J/photon)

Photon # Density (Photon/(m^3 μm))

Area element orthogonal to $\vec{\mathbf{r}}$ (m^2)

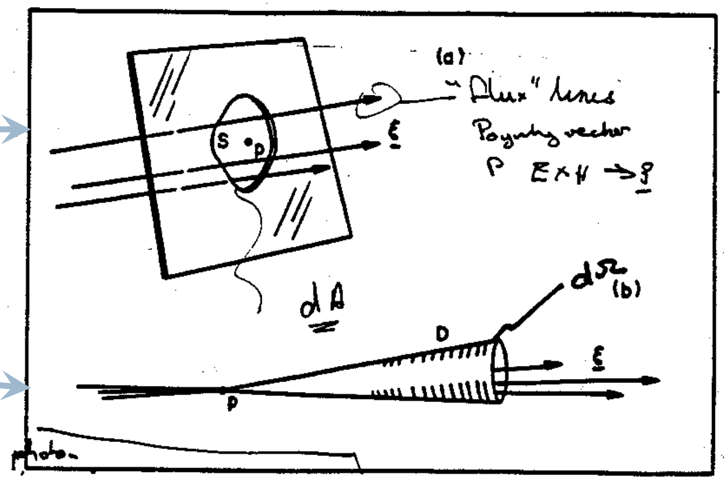
Units = $$\frac{J}{photon} \times \frac{photon}{m^3\ \mu m} \times \frac{m}{s} \times m^2 = \left(J/s\right)\Big/\mu m = \boxed{W/\mu m}$$

Radiance (Intensity)

Two quantities of interest to us follow from $P(\vec{\mathbf{r}})$:

1) Spectral irradiance (flux):

$$F_A = P(\vec{\mathbf{r}})/dA(\vec{\xi}) \quad Wm^{-2}\,(\mu m)^{-1}$$



Recall from above: $<S> = c^2 \varepsilon_o \vec{E} \times \vec{B}$

2) Spectral radiance (intensity):

$$I = P(\vec{\mathbf{r}}) / dA(\vec{\xi})\, d\Omega(\vec{\xi}) \quad W\, m^{-2}\, sr^{-1}\, (\mu m)^{-1}$$

Irradiance: radiant flux of energy passing through area dA

Radiance: radiant flux of energy passing through a point (p) over a small cone of directions defined by $d\Omega$

Again, "spectral" means monochromatic, or "per unit spectral interval" (e.g., unit wavelength, frequency, wavenumber).

Example: How Many Photons?

Here we estimate the rate of photon flow required to deliver a given amount of flux at a specific wavelength. We begin with the definition of spectral flux

$$F_\lambda = \frac{P(s)}{dA} = E \times n(s) \times c,$$

such that

$$n(s) \times c = \frac{F_\lambda}{E} = \frac{F_\lambda}{h\nu} = \frac{F_\lambda \lambda}{hc}$$

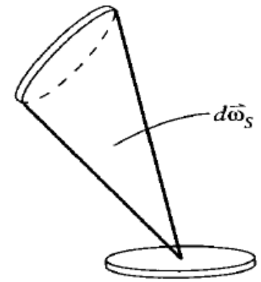
For $F_\lambda = 0.1$ Wm^{-2}μm^{-1}, $\lambda = 0.5$ μm, $c = 3 \times 10^8$ m/s, and $h = 6.63 \times 10^{-34}$ J·s, one obtains

$$n(s) \times c = \frac{\overset{F}{(0.1)}\overbrace{(0.5)(10^{-6})}^{\lambda}}{(6.63 \times 10^{-34})(3 \times 10^8)} = 2.51 \times 10^{17}\ \#/m^2\ \mu m\ s$$

photons of $\lambda = 0.5$ μm flow through a unit area per sec. to produce 0.1 Watts of power per m^2 at that wavelength.

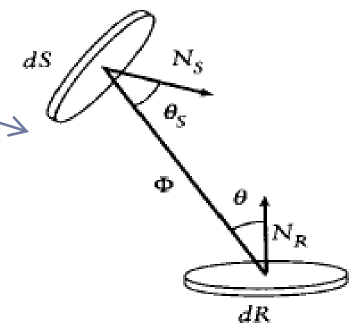
Important Terms Derived from Radiant Flux

Two quantities of interest to us follow from $P(\vec{\mathbf{r}})$:



1) Spectral irradiance (flux):

$F_A = P(\vec{\mathbf{r}})/dA\ (\vec{\xi})$ $\quad$ $W m^{-2}\ (\mu m)^{-1}$



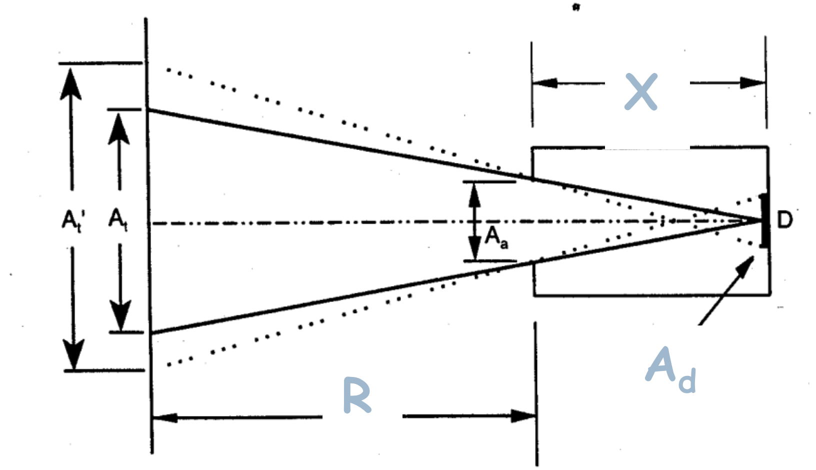
2) Spectral radiance (intensity):

$F_\Omega = P(\vec{\mathbf{r}})\ /\ d\Omega\ (\vec{\xi})$ $\quad$ $W\ sr^{1}\ (\mu m)^{-1}$

Basic quantity measured is the *Radiance:*
$I = P/T$ $\quad$ Units $= W\ m^{-2}\ sr^{1}\ (\mu m)^{-1}$
Where: $\quad T = dA \times d\Omega$
is the instrument *Throughput*, with dA being the area of the detector and $d\Omega$ characterizing its field of view (aperture).

Radiance or intensity is fundamental since we can measure it and all other relevant parameters of interest to us derive from it.

Basic Measurement Concepts
Radiometer



Key Point: radiance is a 'field' quantity being independent of the distance between the instrument and source *assuming the field of view (FOV) is uniformly filled.*

A radiometer is pointed at a uniform wall as shown in the illustration above. The radiant flux received by a detector D of area A_d at the end of a black tube of length X and aperture area A_a as shown is P. Assuming $A_d << X^2$ and $A_a << R^2$, what is the averaged radiance of the wall?

Solution: The solid angle subtended by the entrance aperture at the center of the detector is A_a/X^2

Then, the radiance is given by $P/(A_d A_a/X^2)$

Notes:

(i) Whereas the area of the field of view (target area) is A_t, the total area seen by the whole detector is A_t' which is larger than A_t.

(ii) The radiance is independent of the distance R.

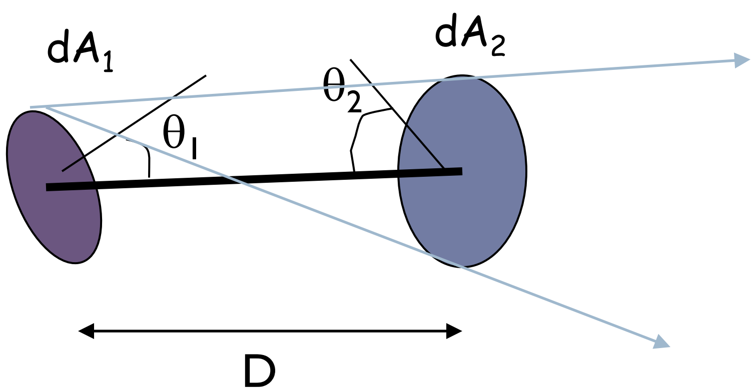
The Concept of Radiance Field Invariance

Solid Angle Subtended by the Aperture

Detector Area

Throughput $$\mathbf{T = d\Omega dA}$$

Radiance $$\mathbf{I = \frac{P}{T}}$$



Radiance is a *field quantity*: it is invariant along a path- this means that it is independent of the distance between the source of radiance and the observer.

Proof:

Consider a source of area dA_1 and a receiver dA_2 separated by D (which is large)

The cross-sections of each end are

$dA_1\cos\theta_1$ and $dA_2\cos\theta_2$

the solid angles subtended by each area

$d\Omega_1 = dA_1\cos\theta_1 / D^2$

$d\Omega_2 = dA_2\cos\theta_2 / D^2$

The throughputs thus follow as

$T_1 = dA_1\cos\theta_1\, d\Omega_2 = dA_1\cos\theta_1\, dA_2\cos\theta_2 / D^2$

$T_2 = dA_2\cos\theta_2\, d\Omega_1 = dA_2\cos\theta_2\, dA_1\cos\theta_1 / D^2$

or

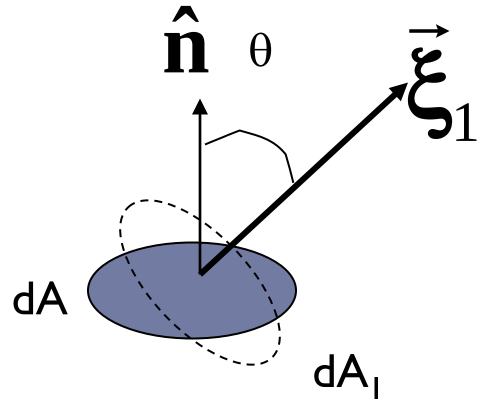
$T_1=T_2$. Thus $I_1=I_2$ since P is constant

The radiance *I* does not vary as the radiation propagates along a path - it is only its eventual interaction with matter that alters *I*.

Converting from Radiance to Irradiance (Flux)

- The radiance along direction $\vec{\xi}_1$ is:

$$\mathbf{I}_1 = \frac{\mathbf{P}_1}{\mathbf{dA}_1\mathbf{d\Omega}} \quad \mathrm{Wm^{-2}\, sr^{-1} (\mu m)^{-1}}$$



- The projection of dA_1 onto the surface perpendicular to $\hat{\mathbf{n}}$ is:

$$\mathbf{dA} = \mathbf{dA}_1 \hat{\mathbf{n}} \bullet \vec{\xi}_1$$

- The total energy through dA per unit area is then:

$$\mathbf{F} = \frac{\mathbf{P}_1}{\mathbf{dA}} = \mathbf{I}_1 \hat{\mathbf{n}} \bullet \vec{\xi}_1 \mathbf{d\Omega}(\vec{\xi}_1) \quad \mathrm{Wm^{-2}\, (\mu m)^{-1}}$$

→ This is called the "irradiance," or "flux"

Converting from Radiance to Irradiance (Flux)

For n sources of radiance $I_{j,}$ j=1…n along the n directions $\vec{\xi}_j$, j=1…n illuminating the surface, the total rate of energy flow per unit area through surface dA is the superposition of each individual source.

$$\mathbf{I}_1 = \mathbf{I}(\vec{\boldsymbol{\xi}}_1)$$

$$F = \frac{P}{dA} = I_1 \hat{n} \bullet \vec{\xi}_1 d\Omega(\vec{\xi}_1) + I_2 \hat{n} \bullet \vec{\xi}_2 d\Omega(\vec{\xi}_2) + \ldots + I_n \hat{n} \bullet \vec{\xi}_n d\Omega(\vec{\xi}_n)$$

$$F = \int I(\vec{\xi}\,') \hat{n} \bullet \vec{\xi}\,' d\Omega(\vec{\xi}\,')$$

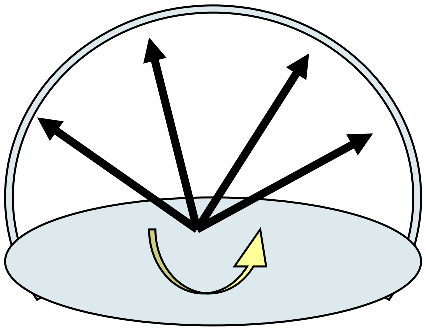
Note that this flux calculation applies to an arbitrarily oriented surface and arbitrary collection of directions (solid angle).

Converting from Radiance to Irradiance (Flux)

Now, consider the special case where the surface is defined in the x-y plane, such that: $\hat{\mathbf{n}} = \hat{\mathbf{k}}$, and using

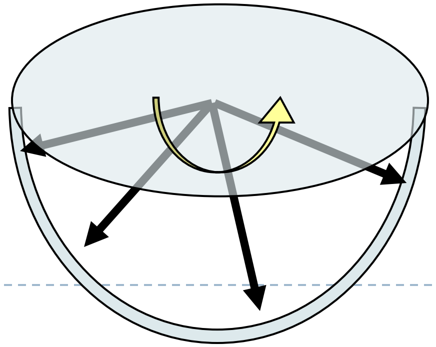
$$\boxed{\begin{aligned}&\vec{\xi} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)\\ &d\Omega = \sin\theta d\theta d\phi\end{aligned}}$$

$$F^{+} = \int_0^{2\pi} d\phi \int_0^{\pi/2} I(\theta,\phi)\cos\theta\sin\theta d\theta$$



$$(\hat{k} \bullet \vec{\xi} = \cos\theta)$$

$$F^{-} = \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} I(\theta,\phi)\cos\theta\sin\theta d\theta$$



Consider the situation where radiation flows onto a surface defined by a discontinuity in refractive index. At the surface

$$\frac{P_1}{A_1} = F_1 = F_2 = \frac{P_2}{A_2}$$

Snell's law predicts that

$$m_1 \sin\theta_1 = m_2 \sin\theta_2 \qquad (\theta_1, \theta_2 \text{ are small by hypothesis})$$

$$m_1\theta_1 = m_2\theta_2$$

$$\theta = \sqrt{\Omega/\pi}$$

and it follows that

$$m_1^2\Omega_1 = m_2^2\Omega_2$$

where we make use of our small cap approximation $\Omega = \pi\theta^2$. Since

$$\Omega_1 I_1 = F_1 = F_2 = \Omega_2 I_2$$

we obtain

$$\frac{I_1}{m_1^2} = \frac{I_2}{m_2^2}$$

Thus, we take I/m^2 as the intensity when we are interested in propagation through an m varying media. The radiance from one m environment to another m environment thus needs to be adjusted by refractive index.

Hemispheric fluxes on a horizontal surface. The upward flux may be defined as

$$\hat{\mathbf{n}} = \hat{\mathbf{k}} \qquad F^+ = \int_0^{2\pi} \int_0^{\pi/2} I(\theta,\phi)\cos\theta \sin\theta d\theta d\phi$$

and the downward flux is

$$F^- = \int_0^{2\pi} \int_{\pi/2}^{\pi} I(\theta,\phi)\cos\theta \sin\theta d\theta d\phi$$

and the net flux is $F = F^+ + F^-$. Often the limits of the θ integral for F^- are flipped, which in turn defines a positive F^- leading to an alternate definition $F = F^+ - F^-$. We will use this *latter* convention throughout. [Note also that the + sign on the upward flux means that the normal of the surface in question points upward along the vertical.]

The intensity and flux from the sun. We will see later how the sun radiates approximately as a blackbody of temperature $T_{\odot} = 5790$ K. This radiation is emitted isotropically from the sun with a broadband intensity (i.e., at an intensity that has been integrated over all wavelengths)

$$I_{\odot} = \frac{\sigma}{\pi} T_{\odot}^4 = 2 \times 10^7 \qquad [\mathrm{Wm^{-2} sr^{-1}}]$$

If we consider the geometry as shown (Fig. 2.6), then the flux from the sun incident on a surface whose normal is along the direction from the point P on the earth's surface to the center of the sun is

$$\begin{aligned} F_{\odot} &= \int_{\Omega_{\odot}} I_{\odot} \cos\theta d\Omega \\ &= I_{\odot} \int_0^{2\pi} d\phi \int_0^{\theta_c} \sin\theta \cos\theta d\theta \\ &= I_{\odot}\pi[\sin^2\theta_c] = I_{\odot}\pi \frac{r_{\odot}^2}{R^2} = I_{\odot}\Omega_{\odot} \\ &\approx 1368 \ [\mathrm{Wm^{-2}}] \end{aligned}$$

(a)

(b)

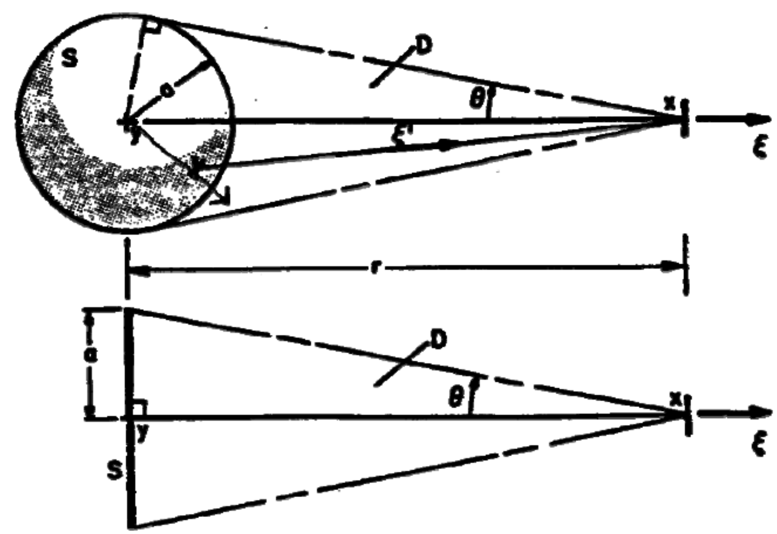


Fig. 2.6 Deriving the irradiance distance-law for spheres and disks.

Example 2.5: The black of night: Olbers' paradox

An ancient astronomer, if asked why the night sky is black, probably would have answered that it was because the sun is absent. If we then ask why the stars don't take the place of the sun, then the likely answer is because the stars are of limited number and individually dim. This last argument has lost its force over the centuries and astronomers tell us that the number of stars occupying the night sky is tremendous indeed. We are left with a paradox of sorts—why is not the night sky as brilliant as the daytime sky filled with the light from an almost infinite number of stars. Olbers pondered this paradox and approached it with the following assumptions:

1. The universe is infinite in extent,
2. The stars are infinite in number, and
3. The stars are of uniform average brightness through all space.

He then considered space as divided into concentric shells about the observer that are large enough to be populated by stars. The amount of light that reaches us from each star (think of this as the product of $I_\odot \Omega_\odot$) varies inversely as the square of its distance from us. But as we look farther out in space the volume of the shell of space also expands (as the distance squared) in such a way that the increased number of stars in the farther shell cancels with the decreased brightness of these more distant stars.

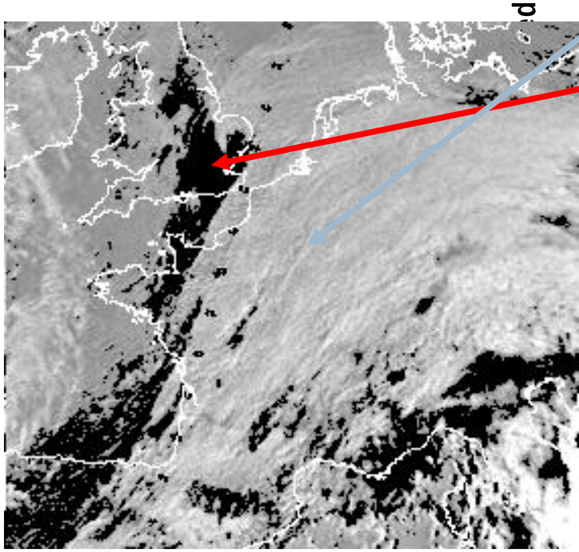
Thus the crux of the paradox is—if the universe is infinite in extent and thus consists of an infinite number of shells, the stars of the universe, however dim they may individually be, ought to deliver an infinite amount of light to Earth. Somewhere in Olbers' paradox there is some mitigating circumstance or logical error. It is commonly thought that the failure of the above arguments occurs with assumption (3). We know that the stars of distant galaxies are receding and this movement caused a red-shift in the spectrum. With the expansion of the universe each succeeding shell delivers less light as it is subject to a successively greater red shift. Thus we receive only a finite amount of energy from the universe and the night sky is black.

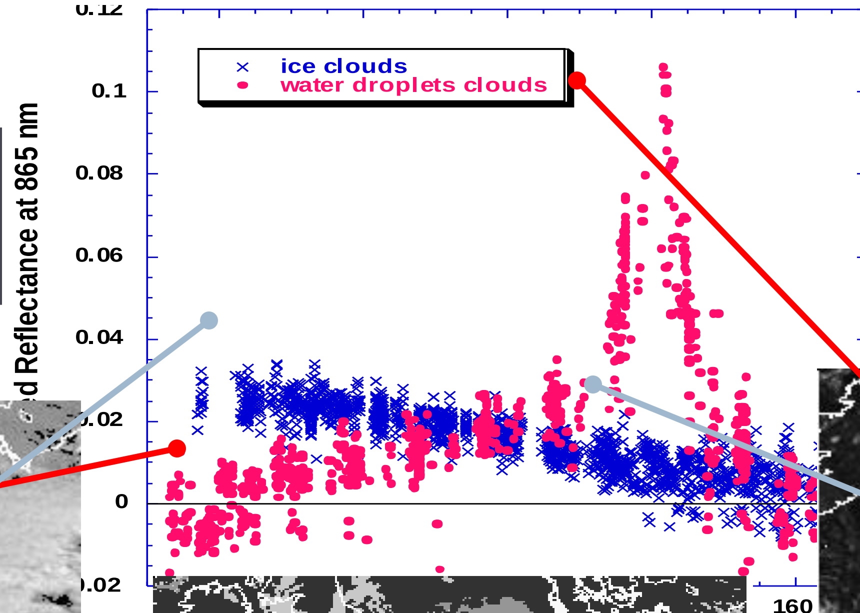
Key Points

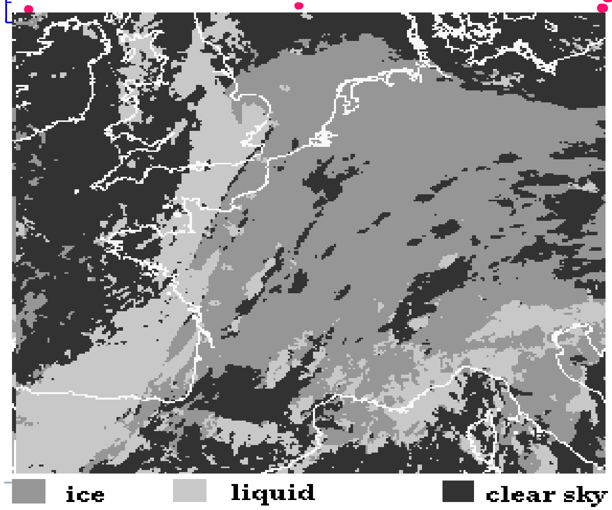
- It is often convenient to consider radiation in a spherical coordinate system.
- The solid angle is a very important term for characterizing radiation emerging from a limited cone of directions.
- Radiance is independent of distance (it is ‘field invariant’) as long as the viewing angle, source, and amount of intervening material are not changed
- All radiometers involve finite-area detectors, and their principal measurement is radiance.
- Irradiance (flux) is the product of radiance and solid angle, and thus decreases inversely to the square of the distance from the radiation source.

Polarization

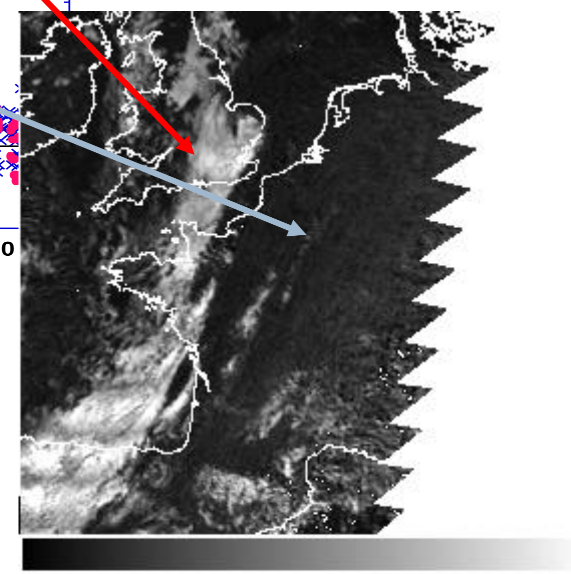
Polarization at 90°
Black = negative polarization





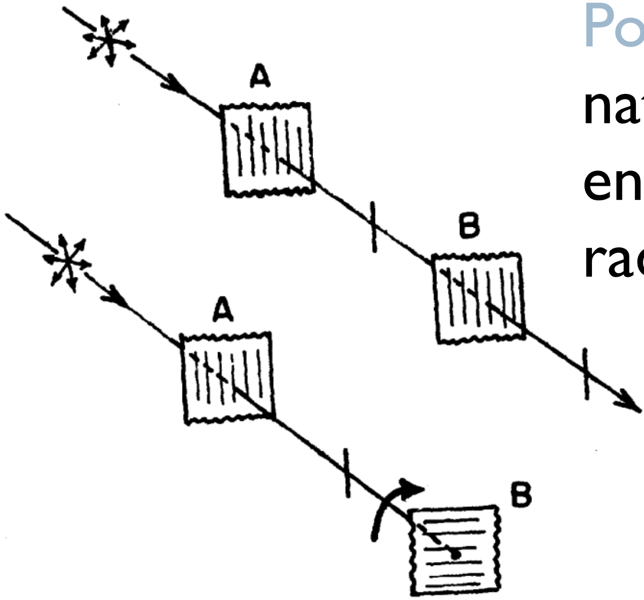


Polarization at 140°



(Parol et al. IEEE, 1999; Goloub et al. JGR, 2000)

Polarization – a property of the transverse nature of EM radiation-doesn't affect energy transfer but it is altered by the way radiation interacts with matter



Simple illustration of the effects of two pieces of polarizing material (polarizers)

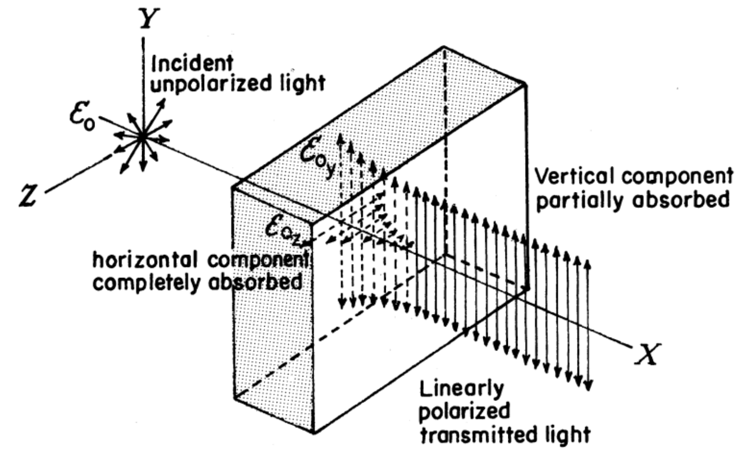
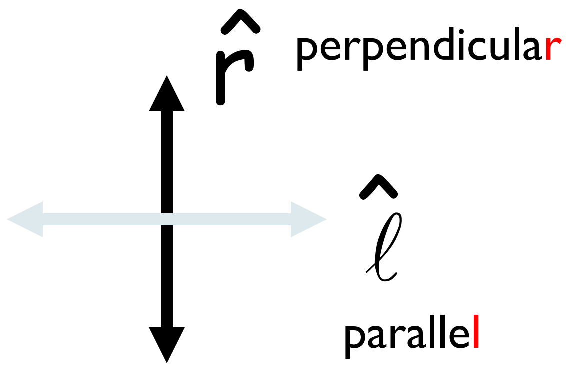
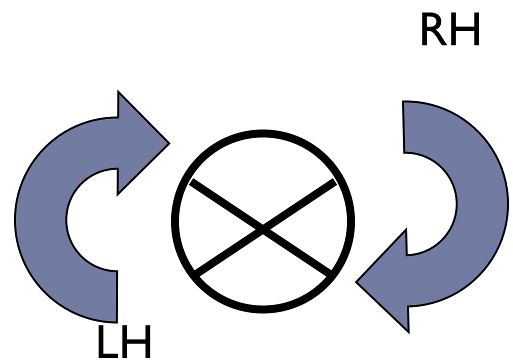


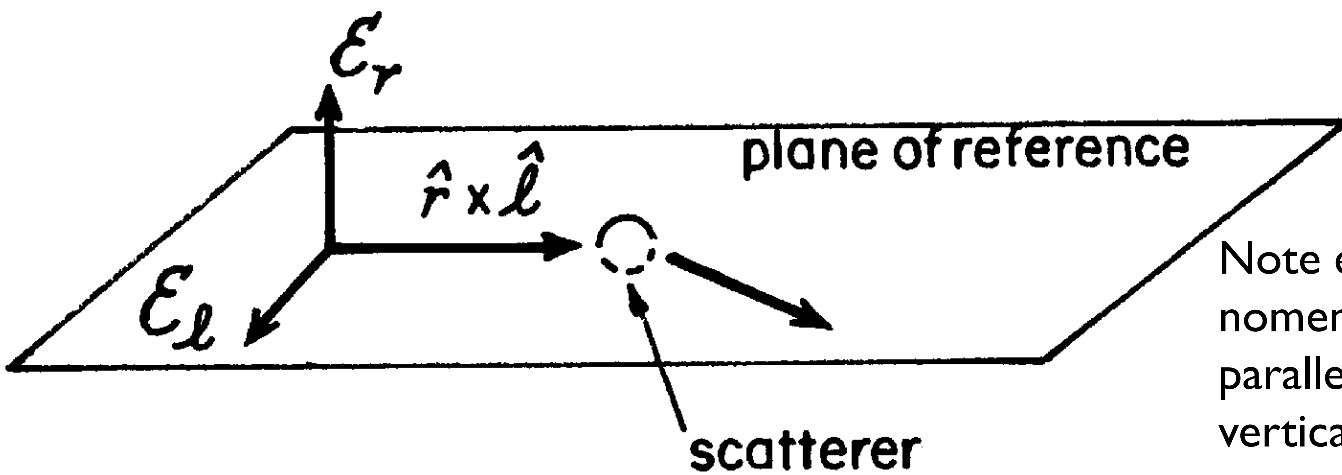
Figure 2.13 An illustration of dichroism.

Polarization is widely used in remote sensing:
- 'multi-parameter' radar → particle characteristics
- microwave emission → cloud water and precipitation
- aerosol
- sea-ice extent
- design of instruments

Mathematical Basis of Polarization



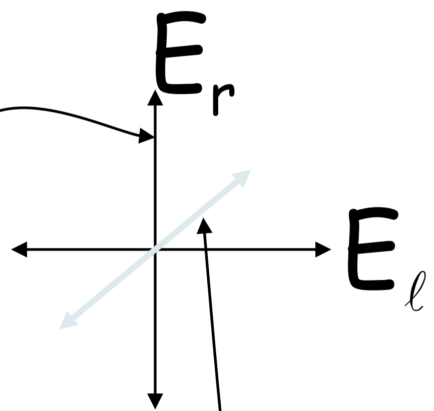




Note equivalent nomenclature –
parallel/perpendicular
vertical/horizontal

Polarization can be mathematically expressed as a superposition of two waves – either two linear wave forms (perpendicular & parallel) or two circular wave forms (left & right rotating waves)

Simple Examples



$$E = E_\ell \hat{\ell} + E_r \hat{r}$$

$$E_\ell = E_{o,\ell} e^{i(\phi+\phi_\ell)}$$

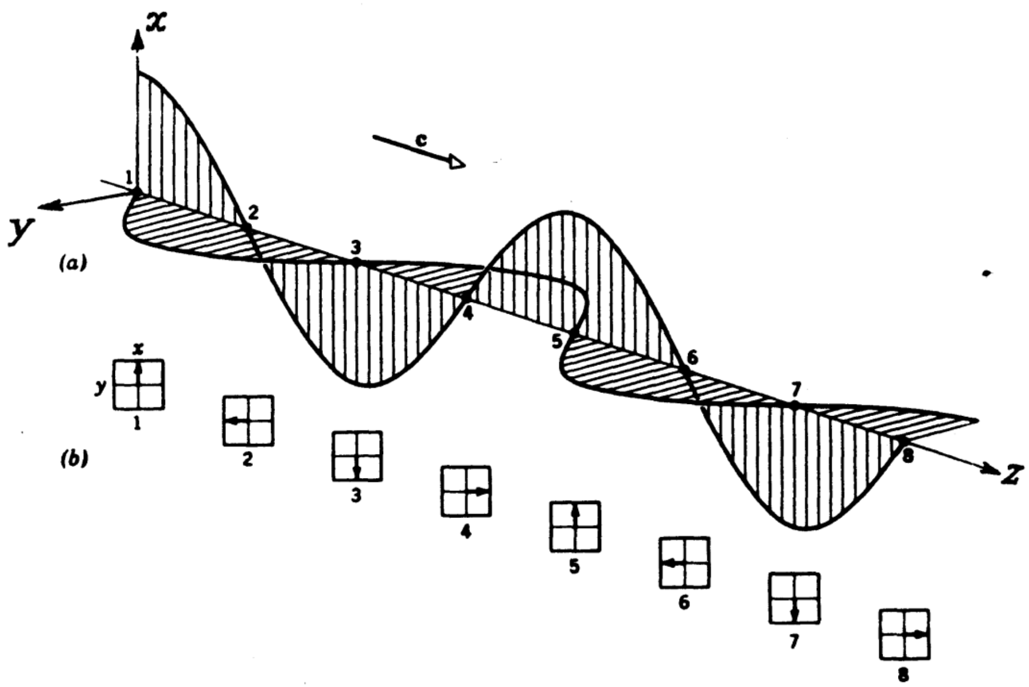
$$E_r = E_{o,r} e^{i(\phi+\phi_r)}$$

Phase difference ϕ_r-ϕ_l and amplitudes define polarization

slope

$$E_r = \frac{E_{o,r}}{E_{o,\ell}} E_\ell$$

Example of linear polarization:
ϕ_r=ϕ_l (oscillate in phase)



Example of circular polarization:

ϕ_r-ϕ_l =π/2 $\quad E_{o,r} = E_{o,\ell}$

Example of unpolarized

ϕ_r-ϕ_l = random $\quad E_{o,r} = E_{o,\ell}$

Note how the sense of the circular polarization depends on sign of phase difference ϕ_r-ϕ_l

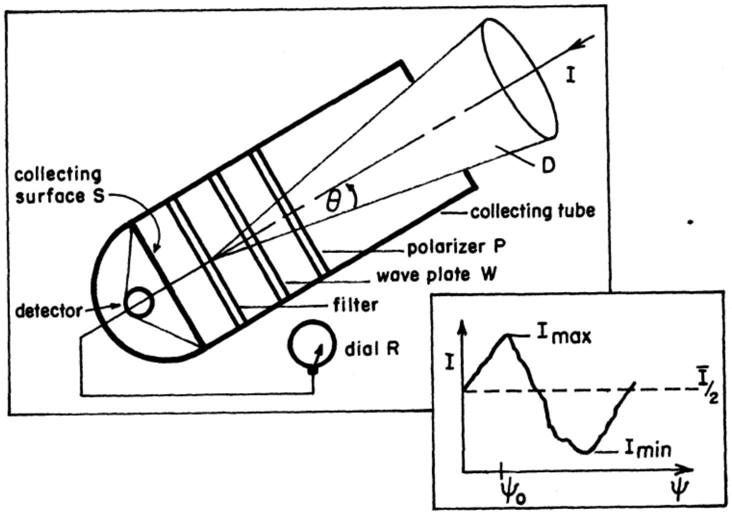
Properties of a wave plate – anisotropic material speed of propagation (refractive index) varies with orientation$\rightarrow$ retardation angle ε

Stokes Parameters – at least two of the the four parameters cannot be measured.

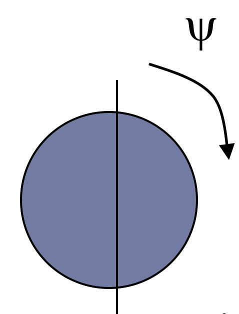
Stokes parameter are an alternate set of 4 intensity (i.e. energy based) parameters that derive directly from experiments

StokesVector

$$\mathrm{I} = \begin{bmatrix} \mathrm{I} \\ \mathrm{Q} \\ \mathrm{U} \\ \mathrm{V} \end{bmatrix}$$



Consider the experimental set up as shown- simple radiometer with a polarizer P and wave-plate W



optical axis

Two experiments - with and without wave plate $\varepsilon = 0$

(i) Signal as a function of rotation ψ

$$\mathrm{I}(\psi, \varepsilon = 0) = \frac{1}{2}[\bar{I} + \Delta I \cos 2(\psi - \psi_o)]$$

$$\bar{I} = I_{\max} + I_{\min}$$

$$\Delta I = I_{\max} - I_{\min}$$

Define

$$\mathrm{Q} = \Delta \mathrm{I} \cos 2\psi_o$$

$$\mathrm{U} = \Delta \mathrm{I} \sin 2\psi_o$$

$$\mathrm{I}(\psi, 0) = \frac{1}{2}[\bar{I} + Q \cos 2\psi + U \sin 2\psi]$$

Stokes Parameters

Table 2.1 Radiance and Stokes Vectors of Common States.

Verbal Description	Observable Intensity Vector	Stokes Vector [I,Q,U,V]
Vertically or parallel polarized intensity	$[1,0,\frac{1}{2},\frac{1}{2}]$	[1,1,0,0]
Horizontally or perpendicular polarized intensity	$[0,1,\frac{1}{2},\frac{1}{2}]$	[1,-1,0,0]
Linearly polarized intensity at +45°	$[\frac{1}{2},\frac{1}{2},1,\frac{1}{2}]$	[1,0,1,0]
Linearly polarized intensity at -45°	$[\frac{1}{2},\frac{1}{2},0,\frac{1}{2}]$	[1,0,-1,0]
Right circularly polarized intensity	$[\frac{1}{2},\frac{1}{2},\frac{1}{2},0]$	[1,0,0,1]
Left circularly polarized intensity	$[\frac{1}{2},\frac{1}{2},\frac{1}{2},1]$	[1,0,0,-1]
Unpolarized intensity	$[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}]$	[1,0,0,0]

$$\text{Degree of polarization} = \frac{\sqrt{Q^2+U^2+V^2}}{I}$$

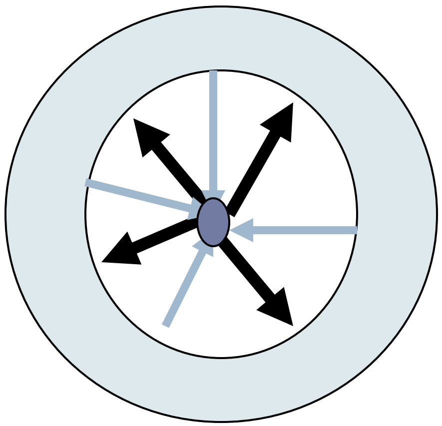
$$\text{Degree of linear polarization} = \frac{\sqrt{Q^2+U^2}}{I}$$

$$\text{Degree of circ polarization} = \frac{V}{I}$$

Basic Laws of Emission

Cavity like – uniform glow of radiation-all objects look the same

Consider an 'isolated cavity' and a hypothetical radiating body at temperature T

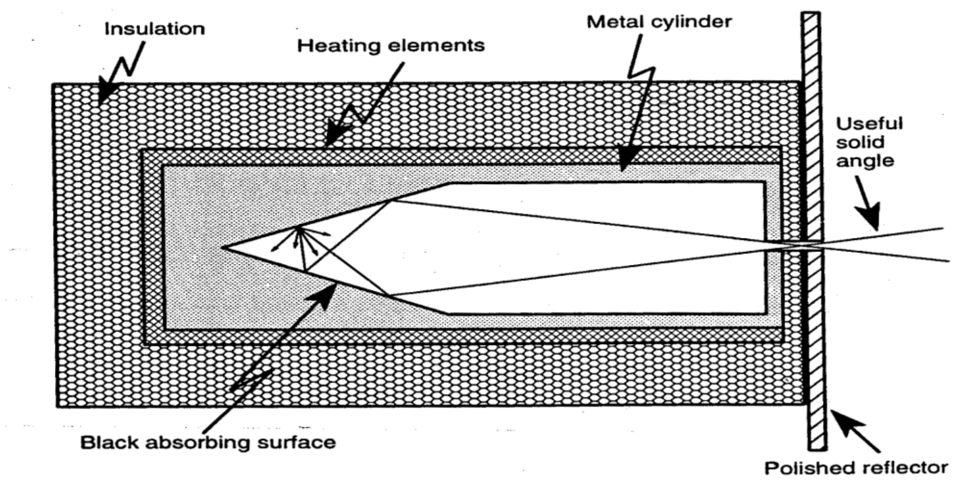


An equilibrium will exist between the radiation emitted from the body and the radiation that body receives from the walls of the cavity

The 'equilibrium' radiation inside the cavity is determined solely by the temperature of the body. This radiation is referred to as black-body radiation.
Two black-bodies of the same temperature emit precisely the same amount of radiation - proof 2nd law

Cavity radiation- experimental approximation to black-body radiation

As we will see below, the amount of of radiation emitted from any body is expressed in terms of the radiation emitted from a hypothetical blackbody. Although the concept of blackbody radiation seems abstract there are a number of very practical reasons to devise ways of creating such radiation. One important reason is to create a source of radiation of a known amount that can be used to calibrate instruments. We can very closely approximate blackbody radiation by carefully constructing a cavity and observing the radiation within it.



Cavities are designed to be light traps - any incident radiation that emerges from the cavity experiences many reflections. If the reflection coefficient of the walls is low, then only a very tiny amount of the energy of incident radiation emerges - most comes from the radiation emitted by the walls of the cavity

Cavities are used both to create a source of blackbody radiation and also as a way of detecting all radiation incident through the cavity aperture.

Khirchoff's law

Suppose that B_{λ}(T) is the amount of blackbody radiation emitted from our hypothetical blackbody at wavelength λ and temperature T. Then

$$\mathbf{E_\lambda = a_\lambda B_\lambda (T)}$$

Kirchoff's law

is the amount of radiation emitted from any given body of temperature T

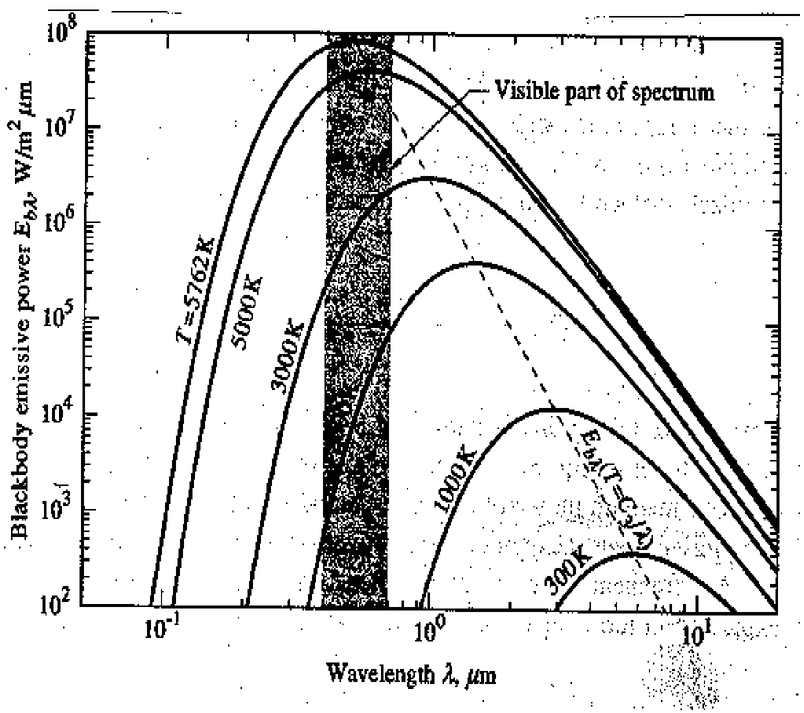
a_λ is the absorptivity or emissivity of the body. Its magnitude and wavelength dependence is solely determined by the properties of the body -such as the composition, and state (gaseous or condensed).

It is often reasonable to suppose that for some bodies a_λ =constant and these are referred to as '*grey bodies*".

a_λ =1 for a black body

Planck's blackbody function

The nature of $B_\lambda(T)$ was one of the great findings of the latter part of the 19th century and led to entirely new ways of thinking about energy and matter. Early experimental evidence pointed to two particular characteristics of $B_\lambda(T)$ which will be discussed later.



$$B_\lambda = \frac{2\pi hc^2}{\pi\lambda^5(e^{hc/\lambda kT} - 1)}$$

$$B_\lambda = \frac{C_1}{\pi\lambda^5(e^{C_2/\lambda T} - 1)}$$

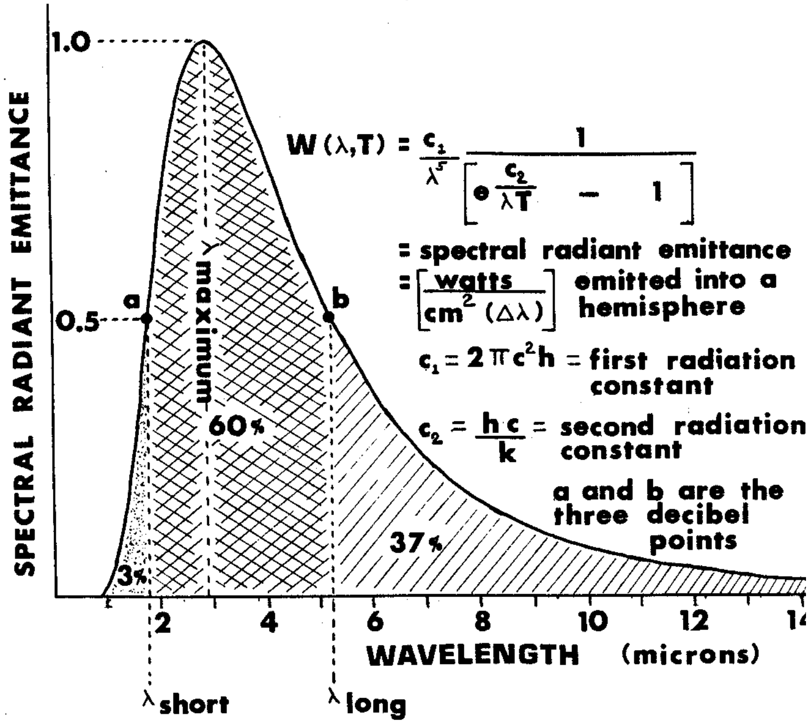
$$C_1 = 2\pi hc^2 = 3.7141832\times10^8\,\mathrm{W\cdot\mu m^4\cdot m^{-2}}$$

$$= 3.7141832\times10^4\,\mathrm{W\cdot\mu m^4\cdot cm^{-2}}$$

$$= 3.7141832\times10^{-4}\,\mathrm{W\cdot nm^4\cdot m^{-2}}$$

$$C_2 = hc/k = 14387.86\,\mu\mathrm{m\cdot K}$$

Derived Characteristics of Planck's Blackbody function



$\lambda_{long} T = 5.1 \times 10^3$ micron-degrees

$\lambda_{short} T = 1.8 \times 10^3$ micron-degrees

Wien Displacement Law

$$\lambda_{max} T = 2898 \mu m \cdot K$$

Example 3.2: At What is the wavelength of the maximum emissive power of the sun? What is the corresponding wavelength of Earth? The temperature of the sun is approximately 5760K and it follows from (3.3) that

$$\lambda_{max} = \frac{2898}{5760} = 0.5\ \mu m$$

which roughly corresponds to the middle of the visible portion of the sectrum. This distribution is very similar to the wavelength distribution of radiation emitted by the sun (Fig. 1.4a). Solar radiation is attenuated as it penetrates the atmosphere. Understanding this attenuation in some detail is one of the goals of this course.

Temperatures of emitters in the Earth's atmosphere vary. Assuming a value of 290K, it follows that

$$\lambda_{max} = \frac{2898}{290} = 10\ \mu m$$

Figure 1.5b is an example of the emission spectrum at the top of the atmosphere measured at one location. This spectral emission does not follow the blackbody curve since it occurs through a kind of transfer from layer to layer in the atmosphere through a combination of absorption at low levels and emission at higher levels and at colder temperatures. The difference between the measured emission and that of a blackbody is crudley indicative of the absorption spectrum of the absorbing gases in the atmosphere. The transfer of radiation and a detailed understanding of the absorption spectrum are topics that we will return to later.

Stefan-Boltzmann Law

$$\pi B(T) = \sigma T^4$$

$B(T=6000)/B(T=300) = 6000^4/300^4$

~160,000

Empirical Radiation Laws

$$B_\lambda = \frac{C_1}{\pi\lambda^5(e^{C_2/\lambda T}-1)}$$

Wien Radiation Law

$$B_\lambda = \frac{C_1}{\pi\lambda^5(e^x-1)}$$

$$e^x >> 1$$

$$B_\lambda \approx \frac{C_1}{\pi\lambda^5 e^{C_2/\lambda T}}$$

Applies at the shorter wavelengths

Rayleigh-Jeans Radiation Law

$$B_\lambda = \frac{C_1}{\pi\lambda^5(e^x-1)}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$$

$$B_\lambda \approx \frac{C_1}{\pi\lambda^5(1+x-1)}$$

$$B_\lambda \approx \frac{C_1}{\pi\lambda^5(x)}$$

$$B_\lambda \approx \frac{C_1}{C_2}\frac{T}{\pi\lambda^4}$$

Breaks down at small wavelengths - the UV catastrophe. This law only applies in the microwave - radiant energy and temperature are synonymous

Lab assignment

1. Using matlab show that $\pi \int_{0}^{\lambda_{\max}} B(\lambda, T)\, d\lambda = \sigma T^{4}$

Brightness Temperature

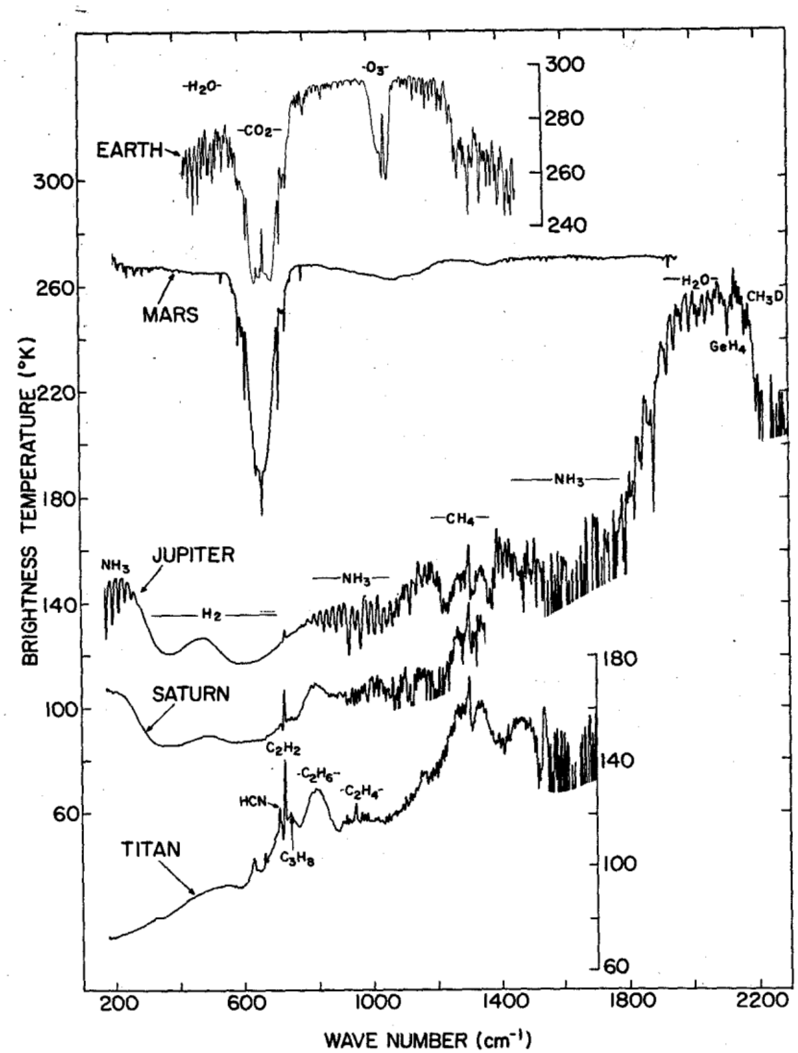
An important temperature of the physical system, and one different from the thermodynamic temperature in general is the temperature that can be attached photons carrying energy at a fixed wavelength. If the energy of such is I_λ, then this temperature is

$$T_\lambda = B^{-1}(I_\lambda) = C_2/\{\lambda \ln[I_\lambda \lambda^5 \pi/C_1 + 1]\}$$

which is referred to as the brightness temperature

The brightness temperature of microwave radiation is proportional in a simple way to microwave radiance:

Rayleigh Jeans Law $\lambda T \rightarrow \infty$ $B(T) \rightarrow kT$

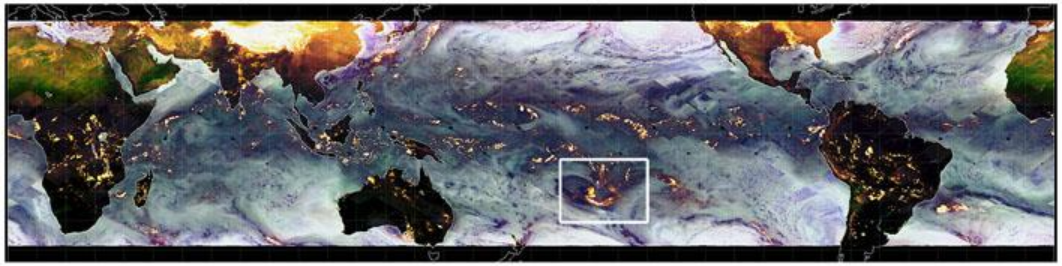


The spectral brightness temperature of planets and moons

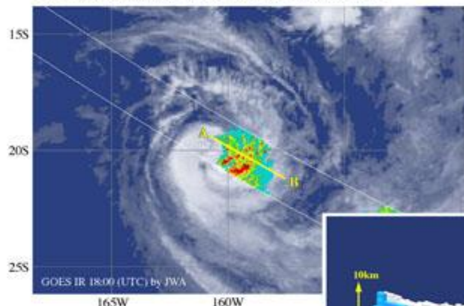
Tropical Rainfall Measuring Mission

First Images: December 8, 1997

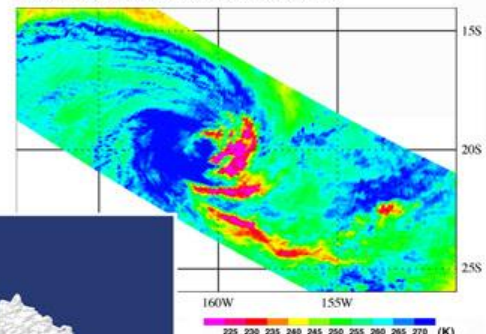
TRMM Microwave Imager (TMI) 2-day (Dec. 7 & 8) composite. Image is a 3-channel combination to highlight cold temperatures (bright yellow) found in many tropical storms. White rectangle identifies Cyclone Pam.

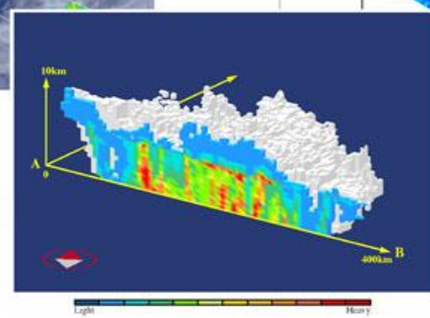


Precipitation Radar (PR) showing rainfall for Cyclone Pam 2 kms. above surface. Cloud image from GOES-09.



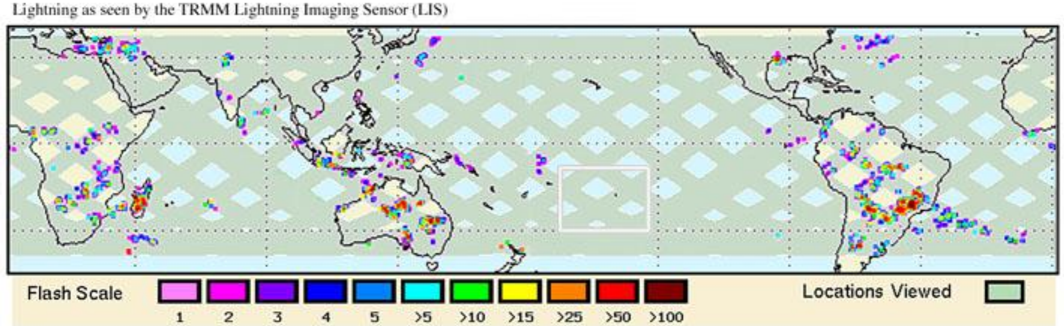
TRMM Microwave Imager (TMI) 85 GHz, horizontally polarized brightness temperature close-up of Cyclone Pam.





3-Dimensional Cross-section of Rainfall from PR Corresponding to Cyclone Pam

Lightning as seen by the TRMM Lightning Imaging Sensor (LIS)



Tropical Cyclone Pam was captured by TRMM in the southern Pacific on the 2nd day of operations for the TRMM Precipitation Radar. Despite the distinctive spiral features seen in the GOES IR, the non-symmetric rainfall pattern about the eye seen in both the PR and TMI indicates that this cyclone is not well organized. No lightning was detected in connection with the cyclone.

Fourier Transforms

- For more information on fourier transforms and its applications you can check: http://www.youtube.com/watch?v=gZNm7L96pfY and all other lectures by Prof Brad Osgood from Stanford University

Basic Points To Master in Radiometry

1. Understand what is meant by *direction* and how it is defined (for this, we need to establish a frame of reference).

2. Understand what is meant by *solid angle* and how we define integrals over it (important for many applications).

3. Understand what is meant by *spectral intensity*, *radiance*, and *hemispheric flux* (a.k.a. *irradiance*).

4. Be able to convert from *radiance* to *irradiance.*

Establishing A Frame of Reference

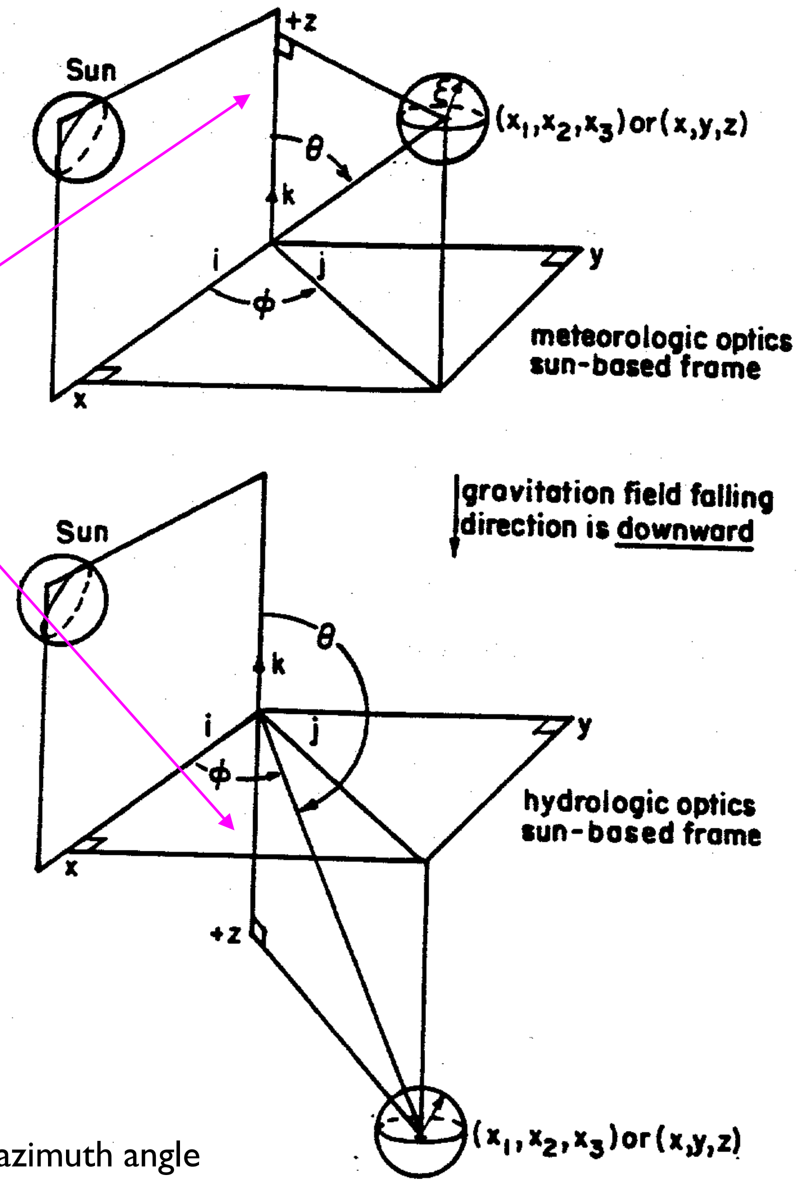
We begin with a Cartesian coordinate system that is orientated such that the sun is contained in the X-Z plane

We will encounter two forms of this 'sun-based' frame of reference:
1) Z-axis increasing upwards (a measure of altitude)
2) Z-axis increasing downwards (a measure of depth—this will come in handy later on).

We can specify any position in this space as given by a general position vector $\vec{\mathbf{r}}$ by the coordinates (X,Y,Z)

The unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are defined along the X-, Y-, and Z-axes

Two fundamental angles of interest to us are:
1) Zenith angle (θ; measured from Z-axis)
2) Azimuth angle (ϕ; measured from X-axis)

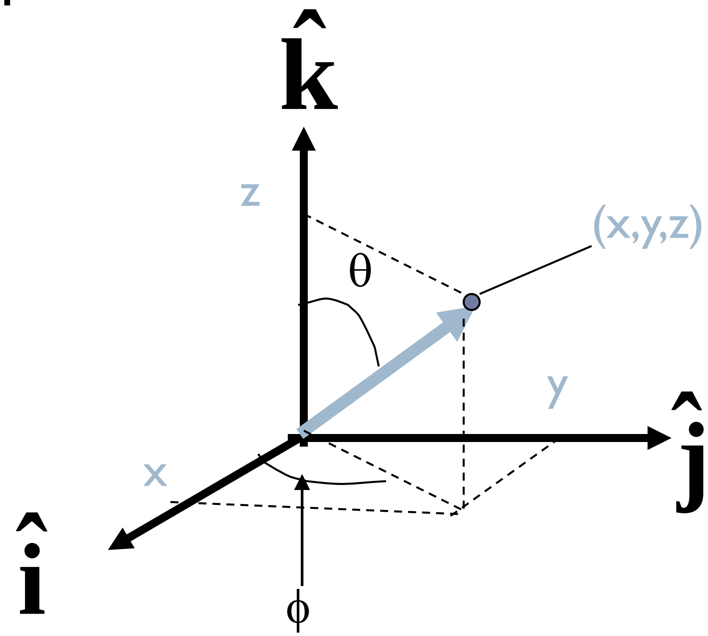


(In satellite meteorology, we are also interested in the relative azimuth angle formed between vectors pointing at the Sun and at the observer)

Defining the Direction Vector

Consider a hypothetical unit sphere about some origin, (0,0,0). We define our *direction vector* as a unit vector that extends from the origin to any point (x,y,z) on that sphere.

$$\vec{\xi} = \frac{\vec{r}}{|\vec{r}|} = \frac{(x, y, z)}{(x^2 + y^2 + z^2)^{1/2}}$$



For a vector of unit length ($|\vec{r}| = 1$):

$$\mathbf{x} = \vec{\mathbf{r}} \bullet \hat{\mathbf{i}} = \cos\phi\sin\theta$$

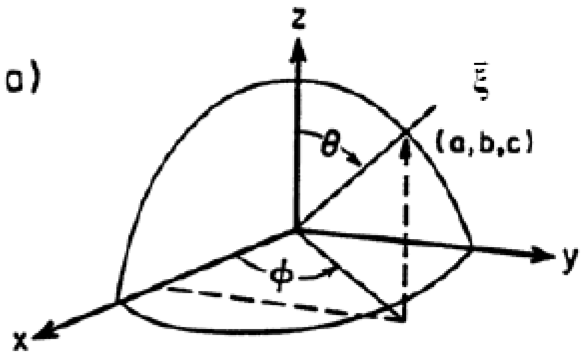
$$\mathbf{y} = \vec{\mathbf{r}} \bullet \hat{\mathbf{j}} = \sin\phi\sin\theta$$

$$\mathbf{z} = \vec{\mathbf{r}} \bullet \overleftarrow{\mathbf{k}} = \cos\theta = \mu$$

We can express a unit direction vector as:

$$\boxed{\vec{\xi} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)}$$

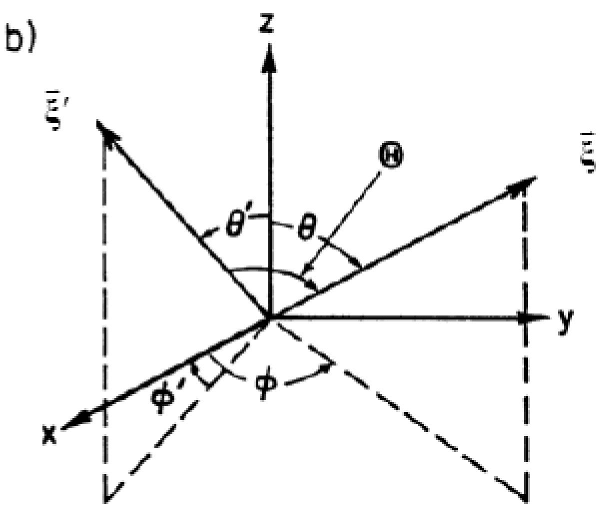
Scattering Angle Between Two Directions



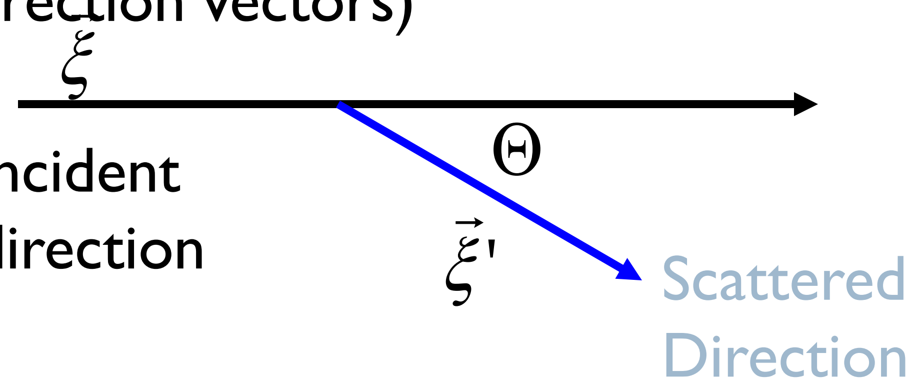
The angle formed between two direction vectors is given by:

$$\cos\Theta = \vec{\xi} \bullet \vec{\xi}'$$

$$= \mu\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\phi' - \phi)$$

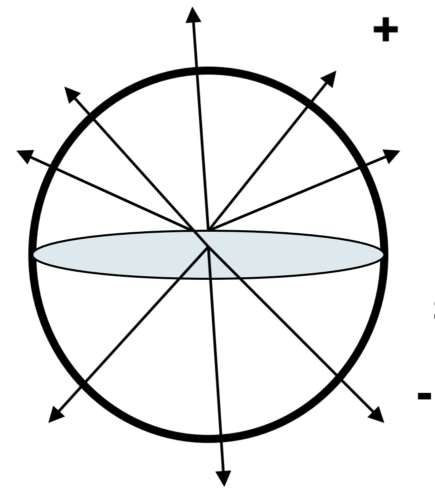


Viewed in 2-D (in a plane that contains the incident and scattered direction vectors)



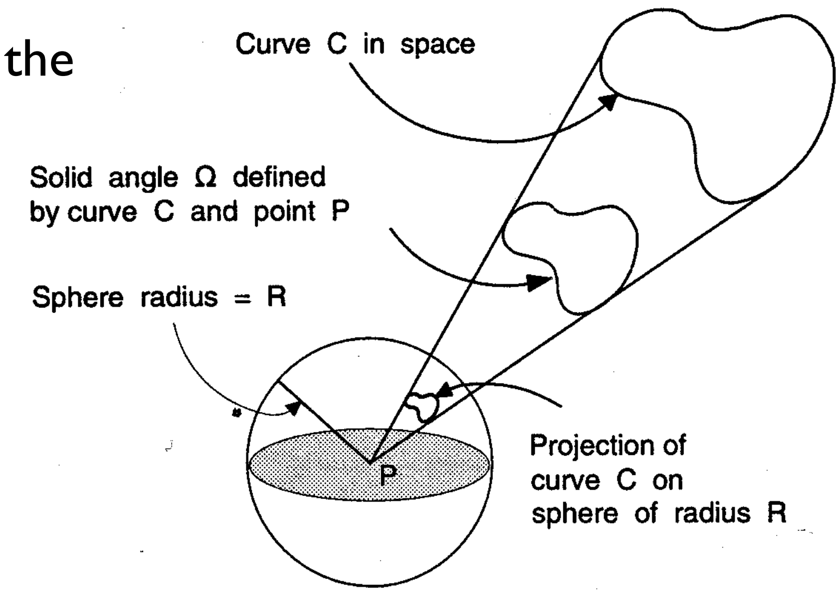
Solid Angle (Ω)

We often wish to characterize the radiation arriving from a particular direction (or, confined to an infinitesimally small "cone" of directions).



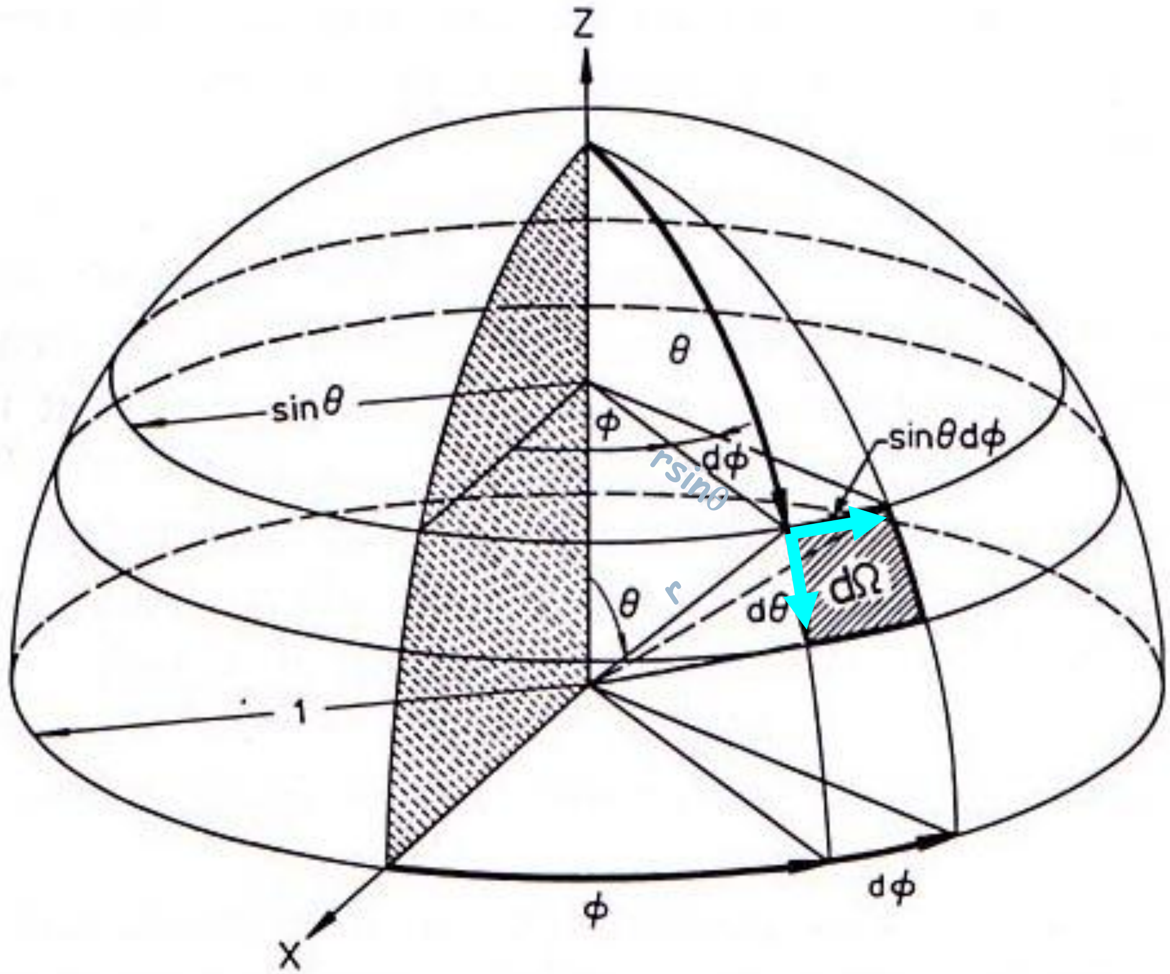
Consider a unit sphere (r=1), established relative to some direction (e.g., local vertical), such that there are two hemispheres (upper and lower)

Now, consider some opening of area dA on the surface of this sphere.



The Solid Angle subtended by the opening as viewed from the origin of the sphere = Area of the opening (dA) on the unit sphere

$$\boxed{\Omega = \frac{A}{r^2}}$$



$$d\Omega = \sin\theta \, d\theta \, d\phi$$

<u>*Units of Solid Angle*:</u>
steradian (sr)

Hemispheric Integrals

We can use the definition of the differential solid angle ($d\Omega = \sin\theta\, d\theta\, d\phi$) to examine the value of solid angle over a hemisphere, or an entire sphere. For a hemisphere:

$$\Omega(hemisphere) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \sin\theta d\theta d\phi$$

$$let\ \mu = \cos\theta \rightarrow d\mu = -\sin\theta d\theta$$

$$\Omega(hemisphere) = 2\pi \int_{\mu=1}^{0} -d\mu = 2\pi \int_{\mu=0}^{1} d\mu = 2\pi(1) = 2\pi$$

(which is simply the surface area of a hemisphere ($A = 4\pi r^2/2 = 2\pi r^2$)

And for a sphere:

$$\Omega(sphere) = 2\pi \int_{\mu=1}^{-1} -d\mu = 2\pi \int_{\mu=-1}^{1} d\mu = 2\pi(2) = 4\pi$$

(which is simply the surface area of a unit sphere ($A = 4\pi r^2$))

Example 2.2: Solid Angle

- The solid angle of a spherical cap defined by the angle θ is

$$\Omega(D) = \int_0^{\theta} \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi[1 - \cos\theta]$$

For small θ, $\cos\theta \to 1 - \theta^2/2 + \ldots$ and

$$\Omega(D) = \pi\theta^2$$

"Small Cap" assumption

For $\theta = -\pi$, the solid angle of a sphere is

$$\Omega(D) = 4\pi.$$

- The solid angle of a spherical segment is

$$\Omega(D) = \int_{\theta_1}^{\theta_2} \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi[\cos\theta_1 - \cos\theta_2]$$

- The solid angle of the sun is

$$\Omega_\odot = \pi\theta^2$$

$\tan(\theta) = \theta + \theta^3/3! + \ldots$h.o.t.

where as we shall see later, $\theta \approx r_\odot / R_{S_E}$, and

$$\Omega_\odot = \pi\left(\frac{0.7 \times 10^6}{1.5 \times 10^8}\right)^2 \approx 0.684 \times 10^{-4} \quad \text{steradian}$$