

# COMPUTATIONAL MODELING IN THE CLASSROOM

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## Abstract

We present two projects on computational modeling with real-world data that can help students develop their mathematical maturity and intuition in the context of purely mathematical concepts. One project is related to the oil leaking from the Deepwater Horizon platform that happened in the Gulf of Mexico in 2010. The other one is on estimating the area of a lung tumor. The projects have been designed to engage prospective and in-service mathematics teachers in exploring computational modeling and problem-solving.

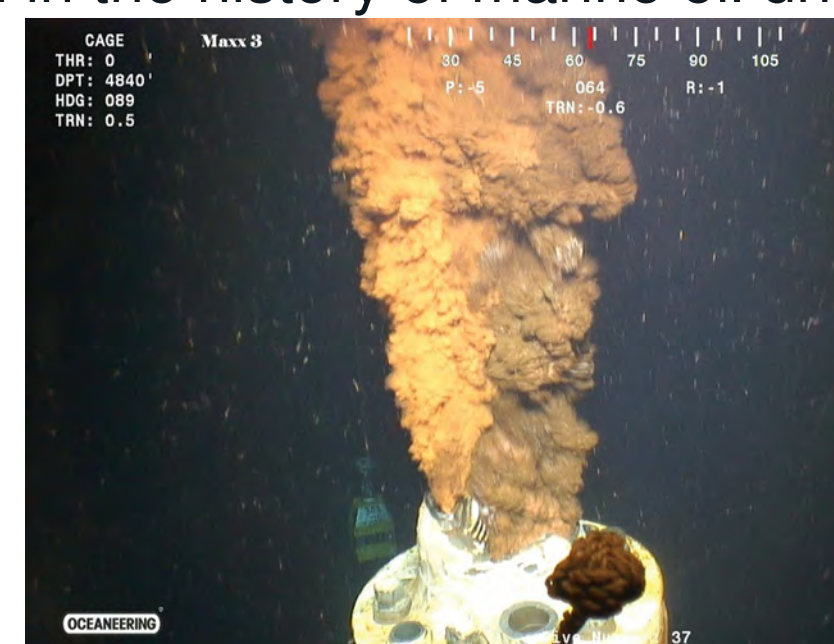
## Motivation

Educators have emphasized the importance of students developing new competencies in understanding and dealing with complex systems constructing, describing, explaining, manipulating, and predicting complex systems; working on multi-component modeling projects in which planning, monitoring, and communicating are critical for success; and adapting rapidly to ever-evolving conceptual tools and new information (Gainsburg, 2006; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

Prospective teachers need to engage with complex and interdisciplinary modeling tasks, modeling related to urgent social issues of the day, conceptualizations and assumptions modeling with technology, critically engaging and interpreting results, communicating conclusions.

## Deepwater Horizon Oil Spill

On April 20, 2010, the oil drilling rig Deepwater Horizon in the Gulf of Mexico, exploded and sank resulting in the death of 11 workers on the Deepwater Horizon and the largest spill of oil in the history of marine oil drilling operations.



A very important question is the following: how much oil in total was leaked during the 87 days of the BP oil spill? It was estimated that around 4.9 million barrels of oil leaked from the damaged well over an 87-day period, before it was finally capped on July 15, 2010. The objective of this project is to use the estimated flow rates over the 87-day oil spill to estimate the total volume of the leaked oil.

The government report cited below provides data for the flow rate of the oil spill at 8 time points, given in the table on the next slide.

The idea is to use these data and create a continuous mathematical function of the flow rate  $flow(t)$  as a function of time  $t$ , which interpolates the data linearly. Then the total volume of the oil leaked during this 87-day period is given by the following:

$$\text{Volume} = \int_1^{87} flow(t) dt$$

We can create a continuous, piece-wise linear function  $flow(t)$  for the flow rate by linearly interpolating the 8 data points, using the 'connector()' function from the **mosaic** R and the developmental version of the **mosaicCalc** R package. First, we need to convert the given data points into a data frame.

Table 1: Flow rate over 8 time points.

Day	Flow Rate
1	0
2	0
3	62000
45	57500
46	59800
84	55200
86	53000
87	0

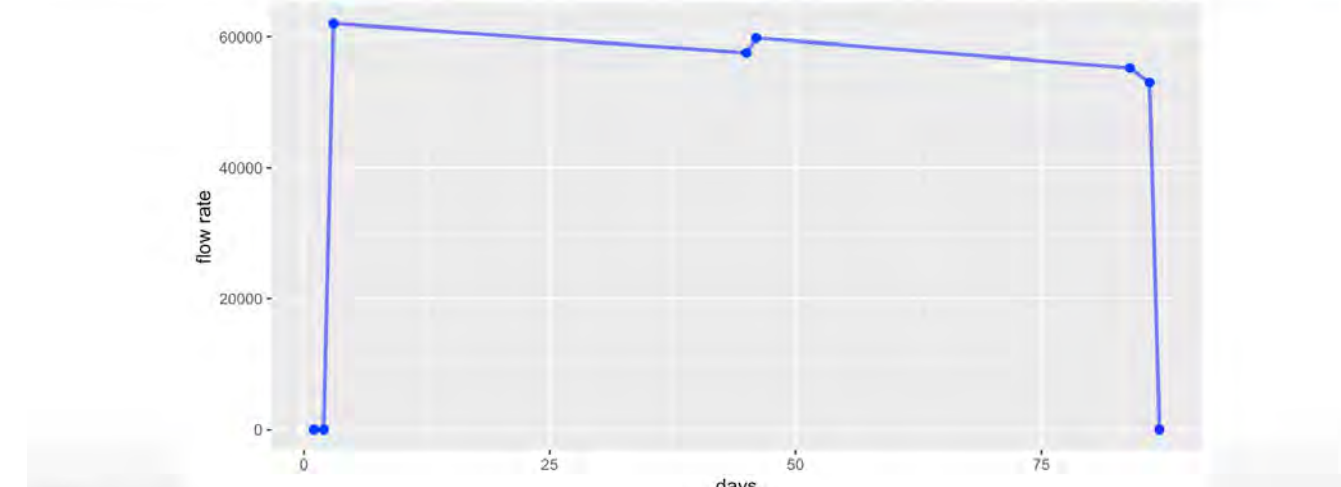
```
# given data points
key_days <- c(1,2,3,45,46,84,86,87)
flow_rates <- c(0,0,62000,57500,59800,55200,53000,0)
flow_data <- data.frame(key_days,flow_rates)
```

We can use 'connector()' to create a piece-wise linear function from the data:

```
flow<-connector(flow_rates ~ key_days, data=flow_data)
```

The 'connector()' returns a function of time in days, named 'flow'. This is the R function that represents the piece-wise linear, continuous mathematical function  $flow(t)$  that connects linearly the data points. We can visualize the given data points along with the interpolating function  $flow(t)$ .

```
gf_point(flow_rates ~ key_days, data=flow_data, size=2, col="blue",
         xlab="days", ylab="flow rate") %>%
slice_plot(flow/day ~ day, domain(day=c(1,87)), size=1, col="blue", alpha=0.6, n=1000)
```



To integrate the flow rate, we compute numerically the anti-derivative function 'volume' of the function  $flow(t)$ , using 'antiD()' from the **mosaic** R package.

```
volume = antiD(flow(t) ~ t)
```

giving us that the total volume of the oil spill is  $4.91885 \times 10^6$  barrels of oil.

The report from the US Department of the Interior gives an estimate of 4.9 million barrels of oil, with estimated uncertainty of  $\pm 10\%$ , so our estimation is very close.

## Estimating the Area of a Tumor

A circular lung nodule needs to be removed and the surgeon needs to know how much tissue to remove. The goal of this project is to estimate the area of the tumor from the X-ray image of the tumor shown earlier.



We have the X-ray image as an png image file, named 'nodule.png'. Thus, we can use the **png** R library to read the png image with 'readPNG()' and plot it on a coordinate system. However, before we can do that we need to convert the image dimensions from pixels into centimeters [cm]. Usually, on a regular X-ray image, one pixel corresponds to  $175 \mu m$  (microns) and since  $1 \mu m = 10^{-4} cm$ , we can approximate the image dimensions in cm using the scaling:

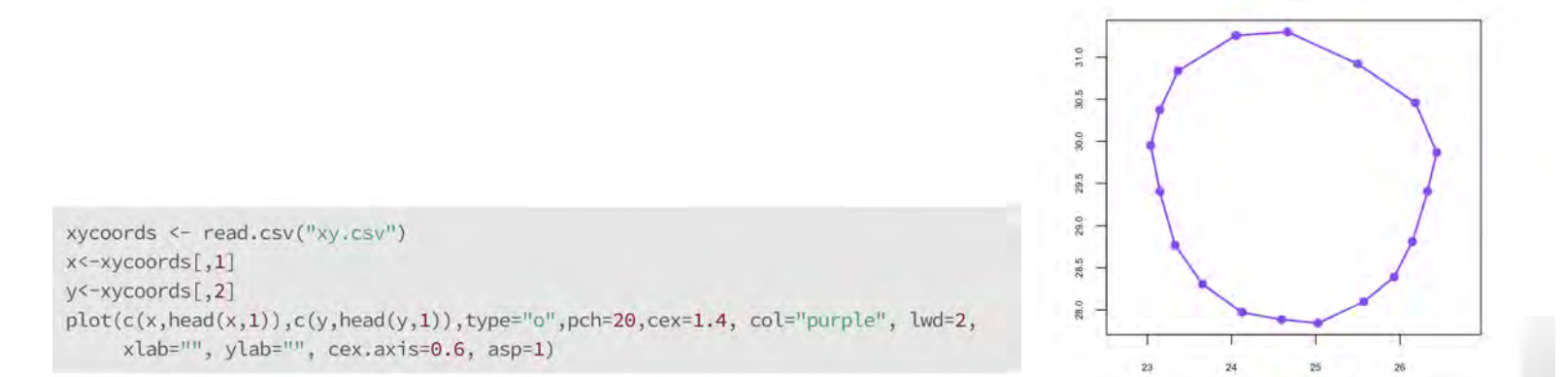
$$\text{pixels} \times 175 \times 10^{-4} [cm].$$

We manually sample 18 points from the tumor boundary using the mouse inside the RStudio Viewer. The manual sampling can be implemented using the 'locator()' function, which is supported only on screen devices such as X11, windows and quartz.

Once we sample the 18 points from the boundary of the tumor, we can plot them on the top of the X-ray image and write the  $x$ -and- $y$  coordinates of the sample points inside a 'csv' file.

```
# sample points inside the RStudio Viewer
coords <- locator(n=18, type="n")
# plot the sampled points from the 1st coords
plot(coords,type="n",col="purple",lwd=3)
# write the x and y coordinates of the sampled points in a csv file
write.csv(coords,file="xy.csv", row.names=FALSE)
```

The 'locator()' function returns a list with the  $x$  and  $y$  coordinates of the sampled points, which are plotted on the top of the X-ray image, specified by the argument 'type="n"'. We can also plot the sampled points only with 'plot(coords)', and we then write the coordinates into the 'xy.csv' file.



The surveyor estimates first the location of the center of the area. From this point we measure the distances to each polygon vertex. Then the distances are averaged to obtain  $\bar{R}$ . One can think of this as an average radius of a circle approximating the area. The area  $A$  is calculated as  $A = \pi \bar{R}^2$ . We can find the center of the polygon by computing its center of mass  $C = (\bar{x}, \bar{y})$ , where  $\bar{x}$  is the average of the  $x$ -coordinates of the polygon vertices, and  $\bar{y}$  is the average of the  $y$ -coordinates of the polygon vertices:

$$\bar{x} = \frac{1}{n} \sum_{k=1}^n x_k, \quad \bar{y} = \frac{1}{n} \sum_{k=1}^n y_k$$

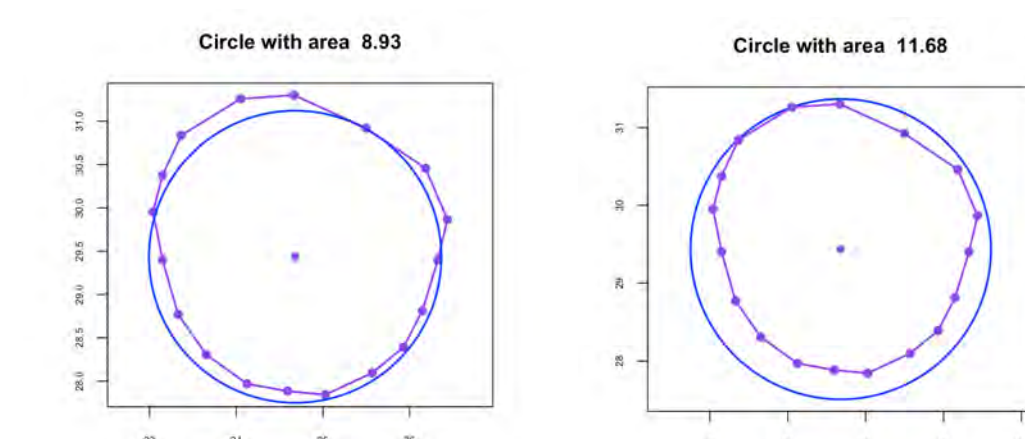
where  $n = 18$  is the number of data points sampled from the tumor boundary. The area of the approximating circle using the average radius is:  $A = \pi \bar{R}^2 = 8.93$ . The area of the approximating circle using the maximum radius is:

$$A = \pi R_{\max}^2 = 11.68.$$

We can plot the circle centered at  $(\bar{x}, \bar{y})$  with the average radius  $\bar{R}$  in polar coordinates:

$$x = \bar{x} + \bar{R} \cos(\alpha), \quad y = \bar{y} + \bar{R} \sin(\alpha), \quad \alpha \in (0, 2\pi)$$

We can similarly plot the approximating circle using the maximum radius.



**Other approaches:** This is a very rich area that gives students the opportunity to explore different approaches and combine mathematical and computational modeling. Other methods that can be explored:

- implement the shoelace formula,
- implement a circle fitting, or
- implement a Monte Carlo simulation for estimating the area.

## References

1. Review of flow rate estimates of the Deepwater Horizon oil spill
2. Report from the US Department of the Interior; assessment of Flow Rate Estimates for the Deepwater Horizon/Macondo Well Oil Spill