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MEDU 2010
HW#2: 0.3: 1,2,3, 0.6: 1,2,3, 0.8: 1,2

0.3.1 Isosceles Triangle Theorem

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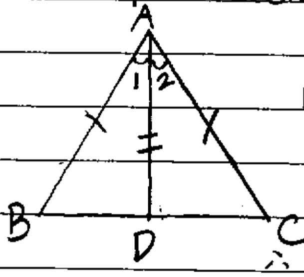

Draw the angle bisector AD.
Then, $m(\measuredangle 1) = m(\measuredangle 2)$
$\overline{AB} \cong \overline{AC}$ (given), $\overline{AD} \cong \overline{AD}$ (reflexive property)
$\therefore$ In $\triangle ABD$ and $\triangle ACD$, $\triangle ABD \cong \triangle ACD$ (SAS)
$\therefore m(\measuredangle B) = m(\measuredangle C)$; or $\angle ABC \cong \angle ACB$

0.3.2

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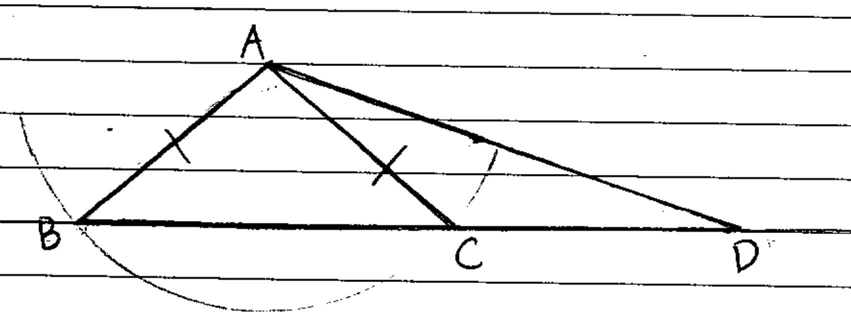

$\triangle ABC$ is an isosceles triangle that $\overline{AB} \cong \overline{AC}$.
$\overline{AD} \cong \overline{AD}$ (reflexive property), $\measuredangle D \cong \measuredangle D$ (reflexive property)
For $\triangle ABD$ and $\triangle ACD$, we have SSA condition.
But, they are not congruent.

0.3.3

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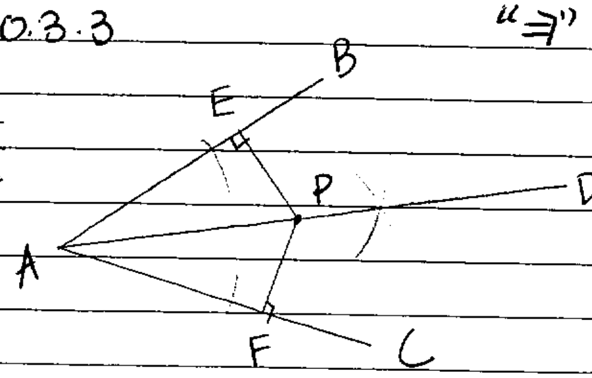

"$\Rightarrow$" $\because$ Point p is in the interior of $\measuredangle BAC$
$\therefore m(\measuredangle BAP) + m(\measuredangle CAP) = m(\measuredangle BAC)$
$\because \overrightarrow{AD}$ bisects $\measuredangle BAC$
$\therefore m(\measuredangle BAP) = m(\measuredangle CAP)$
$\because PE \perp AB$, $PF \perp AC$
$\therefore m(\measuredangle PEA) = m(\measuredangle PFA) = 90^\circ$
$\overline{AP} \cong \overline{AP}$ (reflexive property)
$\therefore \triangle PAE \cong \triangle PAF$ (AAS)
$\therefore \overline{PE} \cong \overline{PF}$

"$\Leftarrow$" $\because$ P is in the interior of $\measuredangle BAC$ and the distance from P to $\overleftrightarrow{AB}$ equals the distance from P to $\overleftrightarrow{AC}$

$\therefore \overline{PE} \cong \overline{PF}$, $PE \perp AB$, $PF \perp AC$

Then, $\triangle APE$ and $\triangle APF$ are right triangles.

$\overline{AP} \cong \overline{AP}$ (reflexive property)

$\therefore \triangle APE \cong \triangle APF$ (HL)

$\therefore m(\measuredangle EAP) \cong m(\measuredangle FAP)$

$\therefore$ The point P lines on the bisector of $\angle BAC$.

0.6.1

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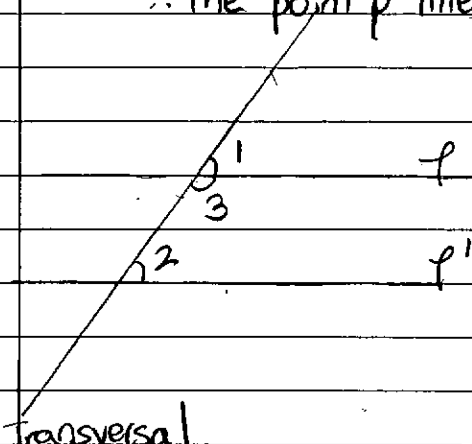

"$\Rightarrow$" $\because$ Two corresponding angles are congruent and cut by the transversal

$\therefore m(\measuredangle 1) = m(\measuredangle 2)$

$\because \measuredangle 1$ and $\measuredangle 3$ are supplementary

$\therefore m(\measuredangle 1) + m(\measuredangle 3) = 180°$

$\therefore m(\measuredangle 2) + m(\measuredangle 3) = 180°$

$\therefore l \parallel l'$ $\leftarrow$ what have you used to conclude this?

"$\Leftarrow$" $\because l \parallel l'$

$\therefore m(\measuredangle 2) + m(\measuredangle 3) = 180°$ $\leftarrow$ and this?

$\because m(\measuredangle 1) + m(\measuredangle 3) = 180°$ (supplementary angles)

$\therefore m(\measuredangle 2) = m(\measuredangle 1)$

$\therefore$ Two corresponding angles are congruent.

0.6.2

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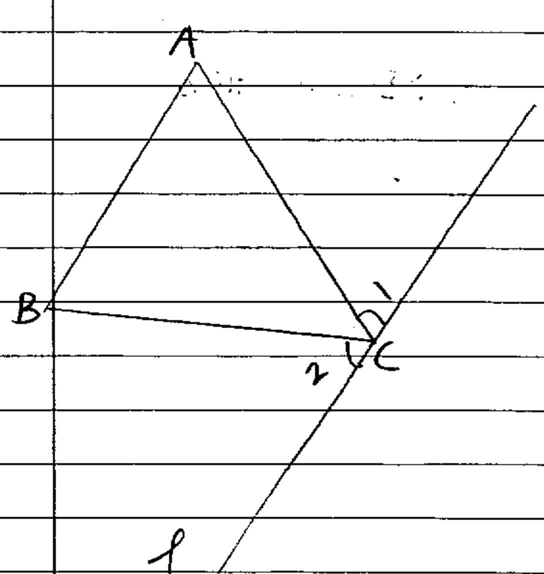


$\because l \parallel AB$

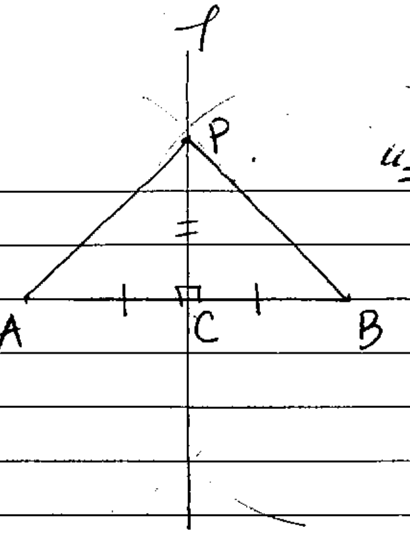
$\therefore m(\measuredangle A) = m(\measuredangle 1)$, $m(\measuredangle B) = m(\measuredangle 2)$ $\leftarrow$ by alt. int. angles Thm

$\because m(\measuredangle 1) + m(\measuredangle 2) + m(\measuredangle C) = 180°$ (linear)

$\therefore m(\measuredangle A) + m(\measuredangle B) + m(\measuredangle C) = 180°$

good, except your notation convention might be misleading – at first I thought $\measuredangle C$ meant something other than the interior angle of the triangle at the vertex C.

0.6.3



"$\Rightarrow$" $\because$ P lies on the perpendicular bisector $\overline{AB}$

$\therefore \overline{AC} \cong \overline{BC}$, $PC \perp AB$ ✓

$\therefore m(\measuredangle PCA) = m(\measuredangle PCB) = 90°$ ✓

$\overline{PC} \cong \overline{PC}$ (reflexive property)

$\therefore \triangle PCA \cong \triangle PCB$ (SAS) ✓

$\therefore \overline{PA} \cong \overline{PB}$ ✓

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"$\Leftarrow$" $\because PA = PB$

$\therefore m(\measuredangle A) = m(\measuredangle B)$ (by exercise 0.3.1)

Assume $\overline{PC}$ is the altitude of $\triangle PAB$, then ← do you just mean - drop the perpendicular from P to $\overleftrightarrow{AB}$?

$m(\measuredangle PCA) = m(\measuredangle PCB)$

$\because \overline{PC} \cong \overline{PC}$

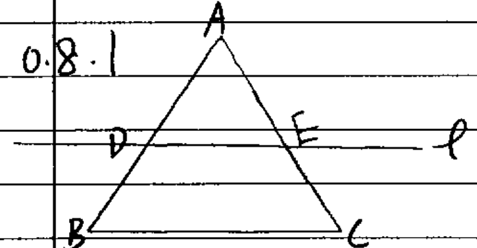
$\therefore \triangle PCA \cong \triangle PCB$ (HL)

$\therefore \overline{AC} \cong \overline{BC}$

$\therefore \overline{PC}$ bisects $\overline{AB}$, then $\overline{PC}$ is the perpendicular bisector

by what theorem?

0.8.1



"$\Rightarrow$" $\because \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

$\therefore m(\measuredangle ADE) = m(\measuredangle ABC)$, $m(\measuredangle AED) = m(\measuredangle ACB)$

$\therefore \triangle ADE \sim \triangle ABC$ (AA) ok

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$

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"$\Leftarrow$" Assume that point D is on $\overline{AB}$ and l passes through D such that $l \parallel \overleftrightarrow{BC}$. From Pasch's axiom, we know that line l also intersects $\overline{AC}$ or $\overline{BC}$.

$\because l \parallel \overleftrightarrow{BC}$

$\therefore l$ intersects $\overline{AC}$ but not $\overline{BC}$ at point E'

$\because l \parallel \overleftrightarrow{BC}$

$\therefore \triangle ADE' \sim \triangle ABC$ and $\dfrac{AD}{AB} = \dfrac{AE'}{AC}$, $\overleftrightarrow{DE'} \parallel \overleftrightarrow{BC}$

$\because \dfrac{AD}{AB} = \dfrac{AE}{AC}$ show more steps here

$\therefore E' = E$, then $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

so $l = \overleftrightarrow{DE}$ and.

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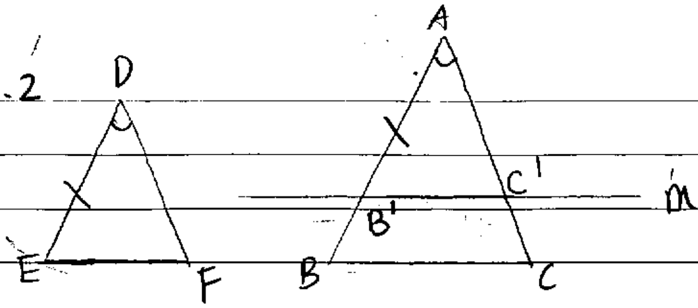
0.8.2



Assume AB > DE

Let B' be the point between A and B such that $AB' = DE$.

Let m be the line through B' that is parellel $\overleftrightarrow{BC}$.

From Pasch's Axiom, line m also intersects $\overleftrightarrow{AC}$ or $\overleftrightarrow{BC}$

$\because$ line $m \parallel \overleftrightarrow{BC}$

$\therefore$ line m intersects $\overleftrightarrow{BC}$ at some point C' but not $\overleftrightarrow{AC}$

$\because$ line $m \parallel \overleftrightarrow{BC}$

$\therefore \triangle AB'C' \sim \triangle ABC$

$\therefore \dfrac{AB}{AC} = \dfrac{AB'}{AC'}$

$\because AB' = DE$

$\therefore \dfrac{AB}{AC} = \dfrac{DE}{AC'}$

$\because \overleftrightarrow{B'C'} \parallel \overleftrightarrow{BC}$, $DE = AB'$

$\therefore DF = AC'$

$\because m(\measuredangle EDF) = m(\measuredangle BAC)$

$\therefore \triangle DEF \cong \triangle AB'C'$ (SAS)

$\because \triangle AB'C' \sim \triangle ABC$

$\therefore \triangle DEF \sim \triangle ABC$