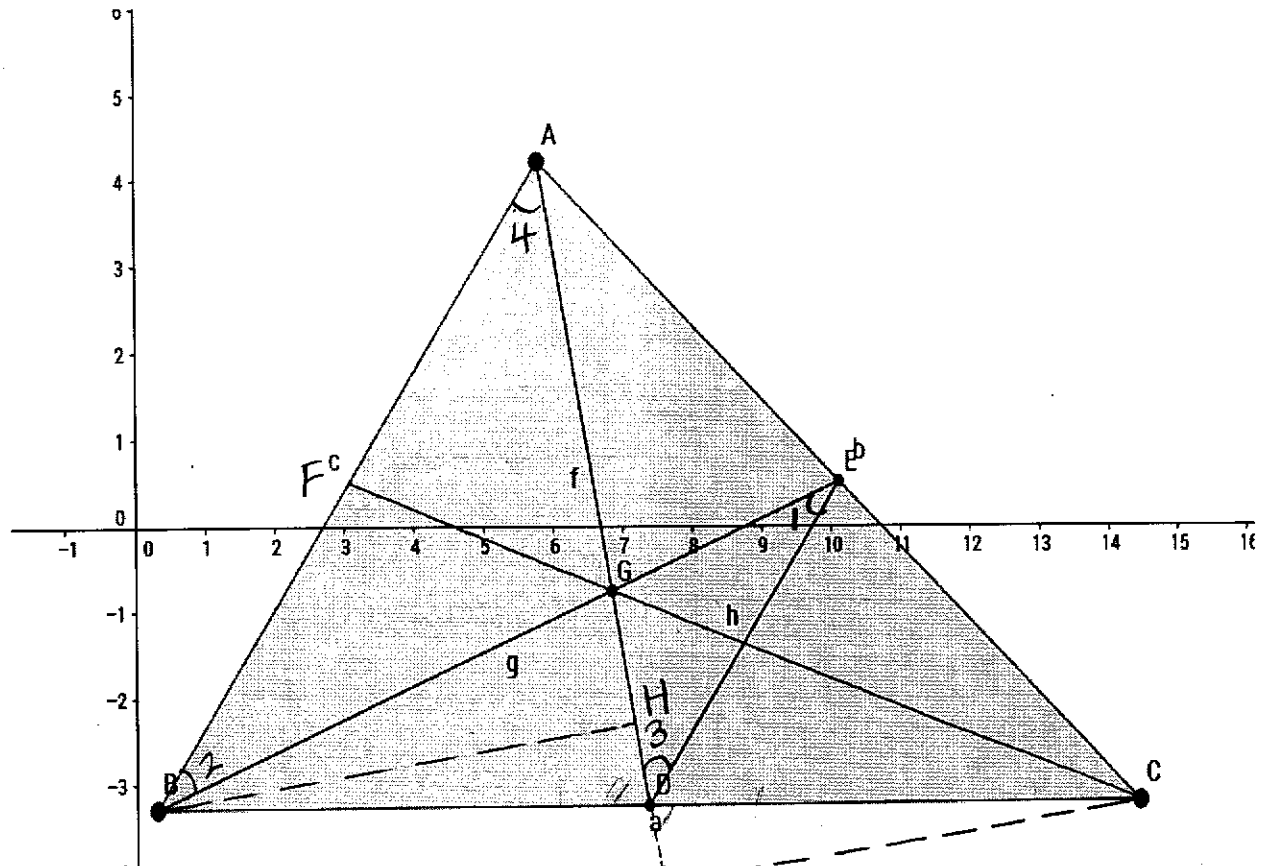


MEDU 2010  
Zhu, Mei

2.2.3-2.2.12



~~scribble~~ ✓

2.2.4

$\therefore$  Points E, D are the midpoints of segments  $\overline{AC}$  and  $\overline{BC}$  accordingly.

$\therefore \overline{ED} \parallel \overline{AB}$  ← why?

$\therefore \overline{ED} \parallel \overline{AB}$

$\therefore m(\angle CED) = m(\angle CAB), m(\angle CDE) = m(\angle CBA)$

Also,  $m(\angle C) = m(\angle C)$

$\therefore \triangle CED \sim \triangle CAB$  (AAA) or (AA)

$\therefore \frac{ED}{AB} = \frac{CE}{CA} = \frac{CD}{CB} = \frac{1}{2}$  (Points E and D are midpoints)

$\therefore \frac{ED}{AB} = \frac{1}{2}, AB = 2ED$

$\frac{3}{4}$

2.2.5

4 From (2.2.4),  $\overline{ED} \parallel \overline{AB}$  or

$\therefore m(\angle 1) = m(\angle 2), m(\angle 3) = m(\angle 4)$  by which reason

$\therefore \triangle ABG \sim \triangle DEG$  (AA)

$\frac{4}{9}$

2.2.6

$\therefore \triangle ABG \sim \triangle DEG$  (Proved in 2.2.5)

$\therefore \frac{GD}{AG} = \frac{GE}{BG} = \frac{ED}{AB} = \frac{1}{2}$  (Proved in 2.2.4)  
almost: put 2.2.4 and 2.2.5 together

$\frac{3}{4}$

$\therefore AG = 2GD, BG = 2GE$ , also  $CG = 2GF$  as G is the centroid and

$\overline{CF}$  is the median

2.2.8

Construct  $\overline{BH} \perp \overline{AD}$  and  $\overline{CK} \perp \overline{AD}$

$\therefore m(\angle BHD) = 90^\circ, m(\angle CKD) = 90^\circ, m(\angle BHD) = m(\angle CKD)$

$$\therefore \alpha(\triangle ADB) = \frac{1}{2} AD \cdot BH, \quad \alpha(\triangle ADC) = \frac{1}{2} AD \cdot CK$$

$$\therefore \alpha(\triangle ADB) = \alpha(\triangle ADC)$$

2.2.9

The centroid is ~~ma~~ formed at the intersection of the medians of the triangle. We call it "the center of mass of the triangle" because the triangle can balance on that centroid point.

2.2.11

Proof.

In  $\triangle ABC$ , point  $G$  is the centroid, and  $AE = EC$ ,  $GE = GE$ , and  $GA = GC$  by the definition of centroid.

$$\therefore \triangle AGE \cong \triangle CGE \text{ (SSS)}$$

Similarly,

$$\triangle AGF \cong \triangle BGF, \quad \triangle BGD \cong \triangle CGD$$

$$\therefore \alpha(\triangle ACG) = \alpha(\triangle ACD) - \alpha(\triangle CGD)$$

$$\alpha(\triangle ABG) = \alpha(\triangle ABD) - \alpha(\triangle BGD)$$

~~$$\alpha(\triangle ACG) = \alpha(\triangle ABG)$$~~

$$\therefore \alpha(\triangle ACG) = \alpha(\triangle ABG), \text{ and } \alpha(\triangle AEG) = \alpha(\triangle EGC), \quad \alpha(\triangle AEG) = \frac{1}{2} \alpha(\triangle ACG)$$

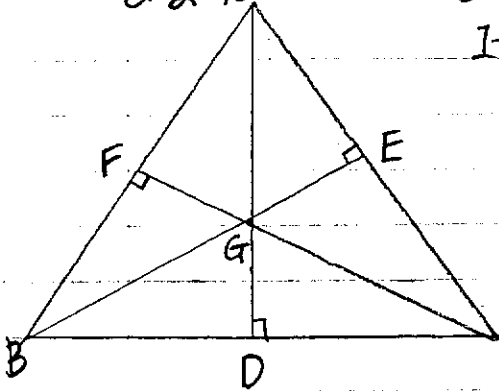
$$\alpha(\triangle AEG) = \frac{1}{2} \alpha(\triangle ACG)$$

$$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG), \quad \alpha(\triangle AFG) = \frac{1}{2} \alpha(\triangle AGB) = \frac{1}{2} \alpha(\triangle ACG)$$

$$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle ECG)$$

$$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle FCG) = \alpha(\triangle AEG) = \alpha(\triangle CDG) = \alpha(\triangle BDG)$$

2.2.12A



Same procedure as in 2.2.11.

If  $\triangle ABC$  is an equilateral triangle, then

$\overline{AD} \perp \overline{BC}$ ,  $\overline{BE} \perp \overline{AC}$ , and  $\overline{CF} \perp \overline{AB}$

$\therefore \triangle ABC$  is an equilateral triangle

$\therefore AB = BC = AC$

$\therefore$  Points D, E, and F are the median points of  $\overline{BC}$ ,  $\overline{AC}$ , and  $\overline{AB}$  accordingly.

$\therefore BD = DC = CE = EA = AF = FB$

Obviously,

$\alpha(\triangle AFG) = \alpha(\triangle BFG)$ ,  ~~$\alpha(\triangle BGD)$~~   $\alpha(\triangle BGD) = \alpha(\triangle CGD)$ ,  
 $\alpha(\triangle GCE) = \alpha(\triangle GAE)$

$\therefore G$  is the centroid of  $\triangle ABC$

$\therefore GD = GE = GF$

$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle BGD) = \alpha(\triangle CGD) = \alpha(\triangle GCE) = \alpha(\triangle GAE)$

Converse, if  $\alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle BGD) = \alpha(\triangle CGD) = \alpha(\triangle GCE) = \alpha(\triangle GAE)$ , then it is no necessary that  $\triangle ABC$  is equilateral.