all form webwork

1. use laplace transform to solve IVP

$y'' + 16y = 48t \quad y(0) = 7 \quad y'(0) = 8$

2. general solution for $y'' + 5xy = 0$ (use series solution) center $x_0 = 0$ $a_0 = 1$ $a_1 = 1$ a

3. inverse transforms laplace of $F(s) = \dfrac{10s^2 + s + 54}{s^3 + 9s}$

4. solve $x^2 y' - 4xy = 8y^2$

5. solve $y'' + 16y' + 64y = 116 + 256t + 128t^2$

1. $\mathcal{L}(y''+16y) = \mathcal{L}(48t)$ formula

$\mathcal{L}(y)(s^2+16) - 8 - 7s = \frac{48}{s^2}$

① $f(t)$ $F(s)$

$t^n \ (n>0)$ $\frac{n!}{s^{n+1}}$ $48t^1 \to 48\cdot\frac{1}{s^2}$

② $\mathcal{L}(y'') = s^2\mathcal{L}(y) - y'(0) - sy(0)$

$Y(s) = \mathcal{L}(y) = \frac{48+7s^3+8s^2}{s^2(s^2+16)}$

$\mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}\mathcal{L}(y) = \mathcal{L}^{-1}\left(\frac{48+7s^3+8s^2}{s^2(s^2+16)}\right) \leftarrow \frac{A}{s^2}+\frac{Bs+C}{s^2+16}$

$A(s^2+16)+(Bs+C)s^2 = 48+7s^3+8s^2$

get $A=3$ $B=7$ $C=5$

$\frac{3}{s^2}+\frac{7s}{s^2+16}+\frac{5}{s^2+16}$

$y(t) = \mathcal{L}^{-1}\left(\frac{3}{s^2}+\frac{7s}{s^2+16}+\frac{5}{s^2+16}\right)$

$y(t) = 3t + 7\cos 4t + \frac{5}{4}\sin 4t$

F(s)	f(t)	
$\frac{n!}{s^{n+1}}$	t^n	$\frac{3}{s^2} = \frac{3\cdot 1!}{s^{1+1}} \to 3t$
$\frac{w}{s^2+w^2}$	$\sin wt$	$\frac{5}{s^2+16} = \frac{\frac{5}{4}\cdot 4}{s^2+4^2} \to \frac{5}{4}\sin 4t$
$\frac{s}{s^2+w^2}$	$\cos wt$	$\frac{7s}{s^2+16} = \frac{7\cdot s}{s^2+4^2} \to 7\cos 4t$

2. $P_0(x) = 1$ converge $(-\infty, +\infty)$ ← $P_0(x)y'' + 5xy = 0$

$y'' + 5xy = 0$

$\sum_{n=0}^{\infty}(n-1)n a_n x^{n-2} + \sum_{n=0}^{\infty} 5a_n x^{n+1} = 0$

$\sum_{n=0}^{\infty}(n+1)(n+2)a_{n+2}x^n + \sum_{n=0}^{\infty} 5a_{n-1}x^n = 0$

$\sum_{n=0}^{\infty}\left((n+1)(n+2)a_{n+2} + 5a_{n-1}\right)x^n = 0$

$\to = 0$

$a_{n+2} = \frac{-5a_{n-1}}{(n+1)(n+2)}$

given $a_0 = 1$

$a_1 = 1$

$a_2 = 0$

$n=1$ $a_3 = \frac{-5a_0}{2\cdot 3} = -\frac{5}{6}$

$n=2$ $a_4 = \frac{-5a_1}{3\cdot 4} = -\frac{5}{12}$

$n=3$ $a_5 = \frac{-5a_2}{4\cdot 5} = 0$

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots$

$= 1 + x + (-\frac{5}{6}x^3) + (-\frac{5}{12}x^4) + \cdots$

3. $f = \mathcal{L}^{-1}(F)$

$= \mathcal{L}^{-1}\left(\frac{10s^2+s+54}{s^3+9s}\right) \leftarrow \frac{A}{s}+\frac{Bs+C}{s^2+9}$

$A(s^2+9)+Bs^2+Cs = 10s^2+s+54$

$A=6$ $B=4$ $C=1$

$\frac{6}{s}+\frac{4s+1}{s^2+9}$

$= \mathcal{L}^{-1}\left(\frac{6}{s}+\frac{4s+1}{s^2+9}\right)$

$= 6 + 4\cos 3t + \frac{1}{3}\sin 3t$

F(s)	f(t)	
$\frac{1}{s}$	1	$\frac{6}{s} \to 6$
$\cos wt$	$\frac{s}{s^2+w^2}$	$\frac{4s}{s^2+9} = \frac{4\cdot s}{s^2+3^2} \to 4\cos 3t$
$\sin wt$	$\frac{w}{s^2+w^2}$	$\frac{1}{s^2+9} = \frac{\frac{1}{3}\cdot 3}{s^2+3^2} \to \frac{1}{3}\sin 3t$

4. $x^2y' - 4xy = 8y^2$ Bernoulli $y' + p(x)y = f(x)y^r$

$y' - \frac{4}{x}y = \frac{8}{x^2}y^2$

$p(x) = -\frac{4}{x}$ $f(x) = \frac{8}{x^2}$ $r=2$

① $y' + p(x)y = 0$

$y' - \frac{4}{x}y = 0$

$y = Ce^{-\int p(x)dx}$

let $C=1$ $y_1 = e^{\int \frac{4}{x}dx}$

$y_1 = e^{4\ln|x|}$

$y_1 = e^{\ln x^4}$

$y_1 = x^4$

② $\frac{u'}{u^r} = f(x)y_1^{r-1}$

$\frac{u'}{u^2} = \frac{8}{x^2}\cdot x^4$

$\int u^{-2}du = \int 8x^2dx$

$-u^{-1} = \frac{8}{3}x^3 + C$

$\frac{1}{u} = -\frac{8}{3}x^3 + C$

$u = -\frac{3}{8x^3} + C$

③ $y = uy_1$

$= -\frac{3x}{8} + Cx^4$

5、2nd order nonhomogeneous

$y'' + p(x)y' + q(x)y = f(x)$

$p(x) = 16 \quad q(x) = 64 \quad f(x) = 116 + 256x + 128x^2$

$y'' + 16y' + 64y = 116 + 256x + 128x^2$

① $y'' + 16y' + 64y = 0$

$r^2 + 16r + 64 = 0$

$(r+8)^2 = 0$

$r = -8$

$y_1 = e^{-8x} \quad y_2 = xe^{-8x}$

② $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$ plug in

$2A + 16(2Ax+B) + 64(Ax^2+Bx+C) = 116 + 256x + 128x^2$

get $A = 2$

$B = 3$

$C = 1$

$y_p = 2x^2 + 3x + 1$

③ $y = y_p + C_1y_1 + C_2y_2$

$= 2x^2 + 3x + 1 + C_1e^{-8x} + C_2e^{-8x}x$